

NeuroImaging Data Processing

aka. Statistical Parametric Mapping short course

Course 6:

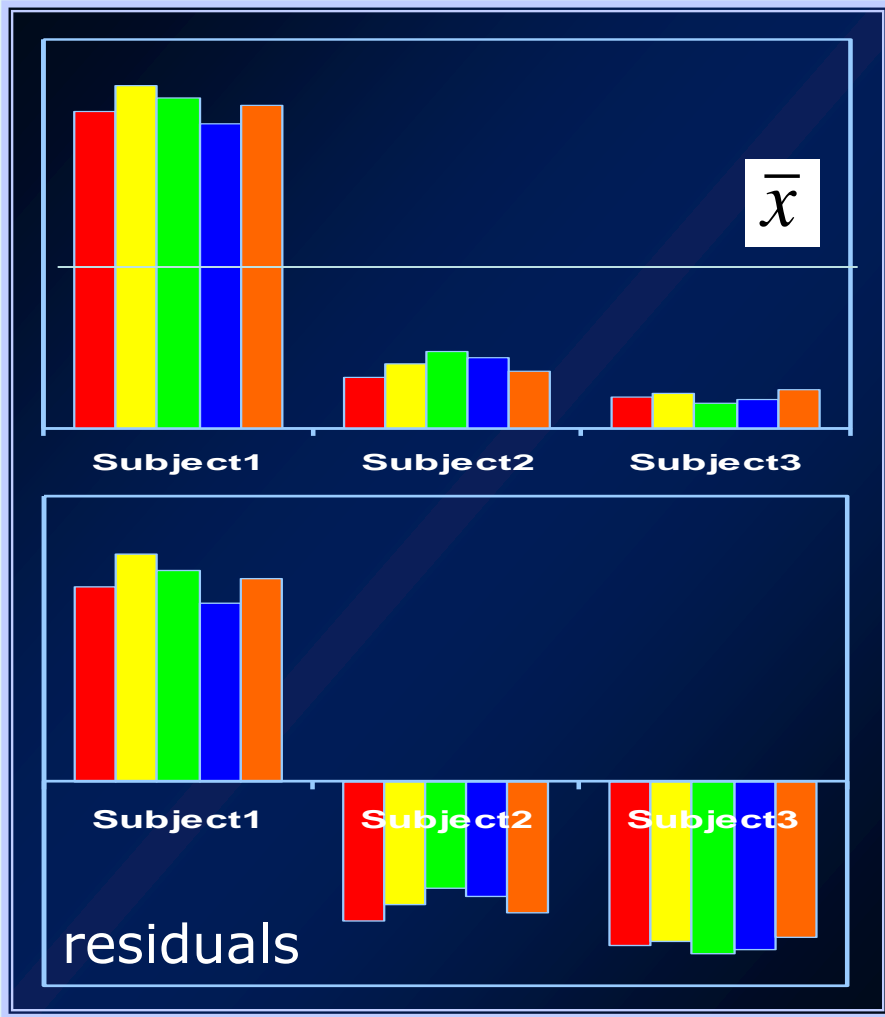
Random Effect Analysis

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GIGA – CRC *In Vivo* Imaging &

GIGA – *In Silico* Medicine

Random effects & variance components



- Fixed effects
 - Are you confident that a new observation from any of subjects 1-3 will be around their mean?
 - Yes! using *within-subjects variance*
 - infer for these subjects – *case study*
- Random effects
 - Are you confident that a new observation from a new subject will be around the mean of first 3?
 - No! using *between-subjects variance*
 - infer for any subject – *population*

Random Effects Illustration

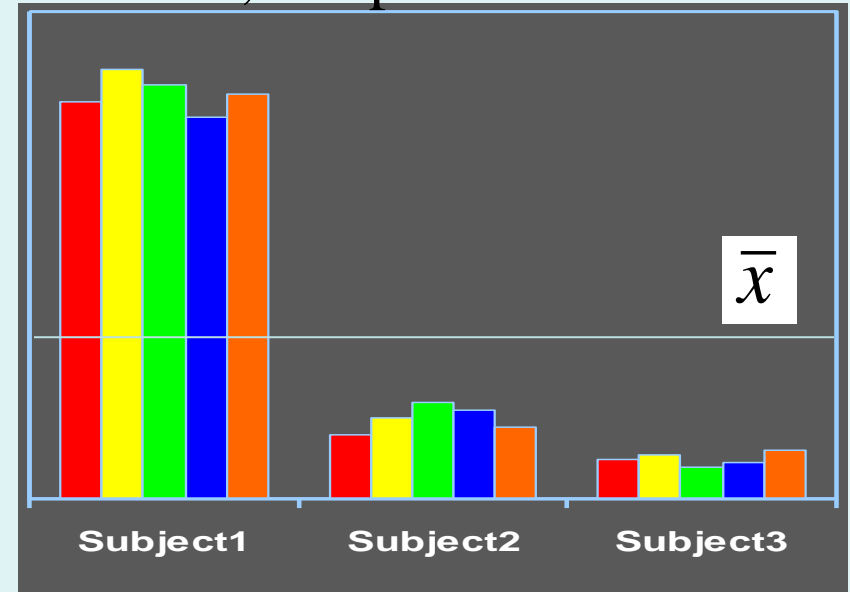
- Standard linear model

$$Y = X\beta + \varepsilon$$

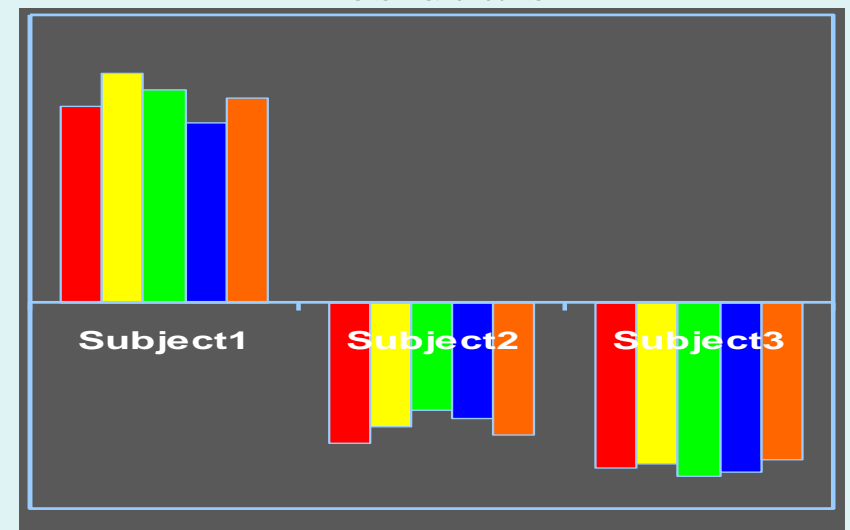
assumes only one source of *iid* random variation

- Consider this RT data
- Here, two sources
 - Within subject var.
 - Between subject var.
 - Causes dependence in ε

3 Ss, 5 replicated RT's



Residuals



Fixed vs. Random effects

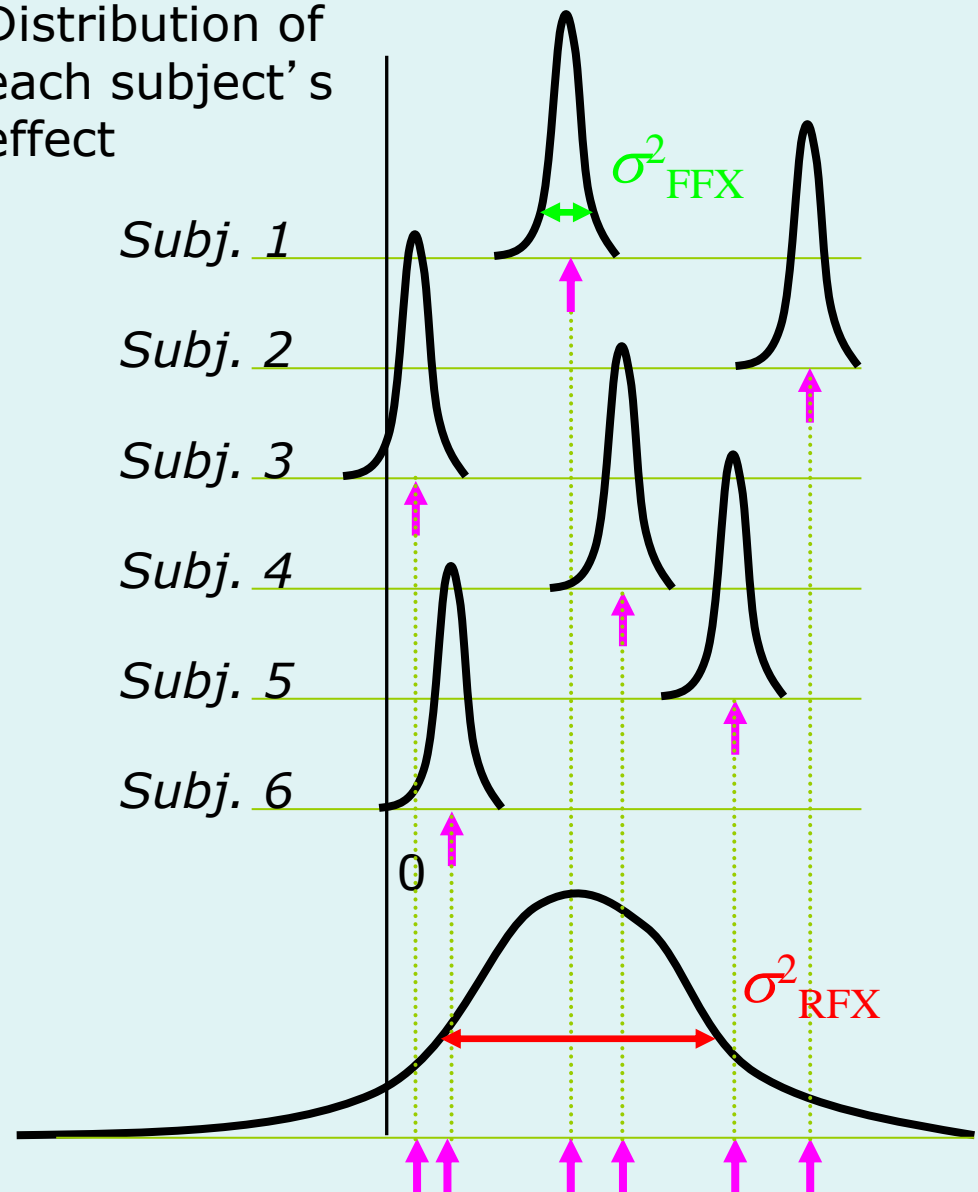
- Fixed Effects

Intra-subject variation suggests *all these subjects* different from zero

- Random Effects

Inter-subject variation suggests *population* not very different from zero

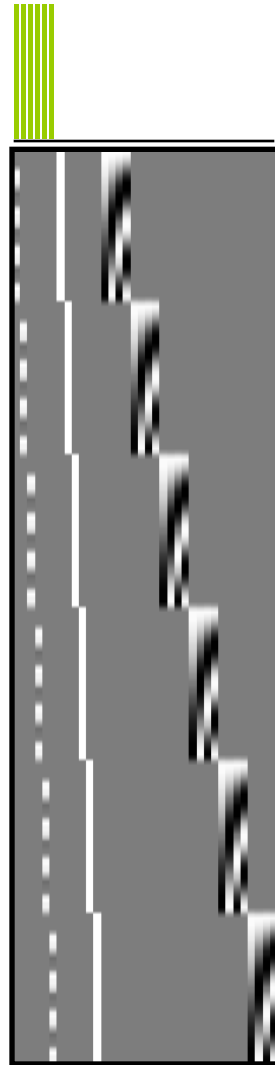
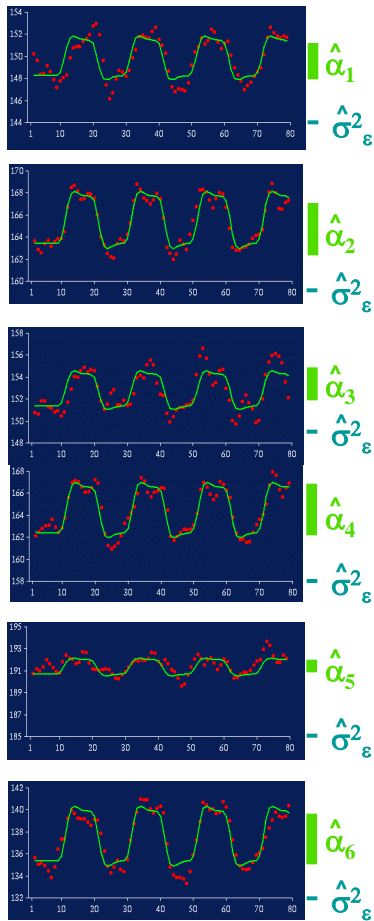
Distribution of each subject's effect



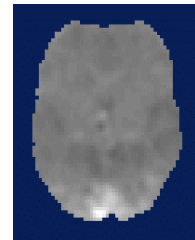
Fixed vs. Random

- Fixed is not “wrong,” just usually is not of interest
- Fixed Effects Inference
 - “I can see this effect in this cohort”
- Random Effects Inference
 - “If I were to sample a new cohort from the population I would get the same result”

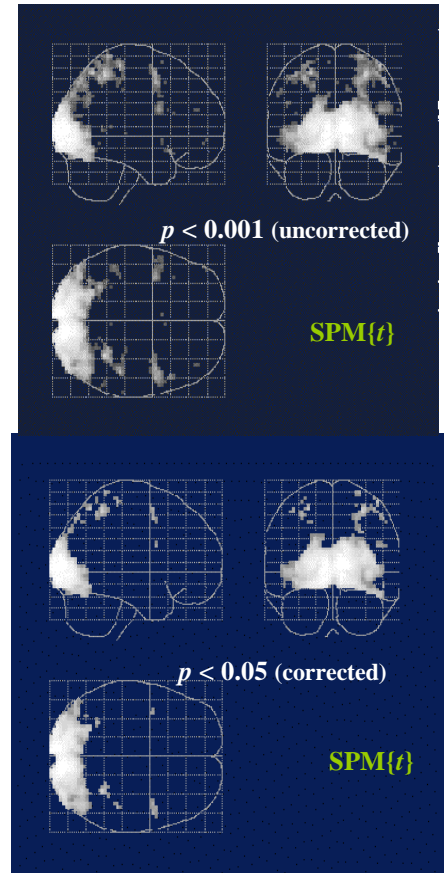
Multi-subject analysis...?



**estimated mean
activation image**

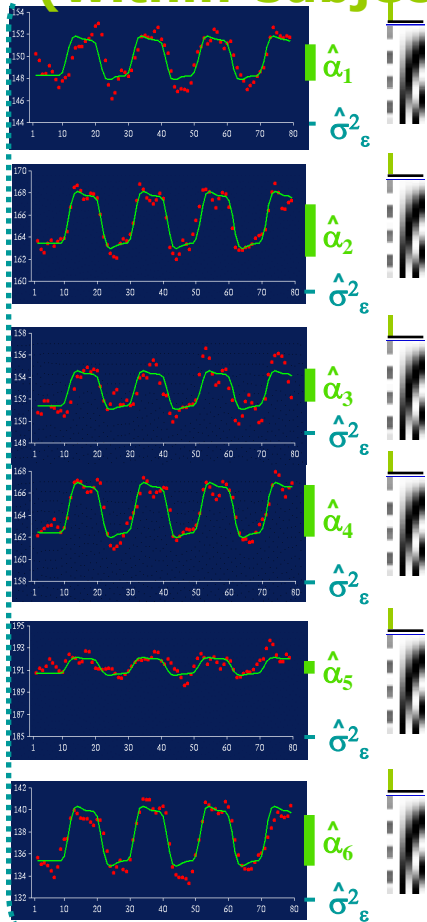


$\overline{\hat{\alpha}_\cdot}$ – c.f. $\sigma_\varepsilon^2 / nw$
 $\hat{\sigma}_\varepsilon^2$ – c.f. -



Two-stage analysis of random effect...

level-one (within-subject)



timecourses at [03, -78, 00] contrast images

level-two (between-subject)

variance $\hat{\sigma}^2$

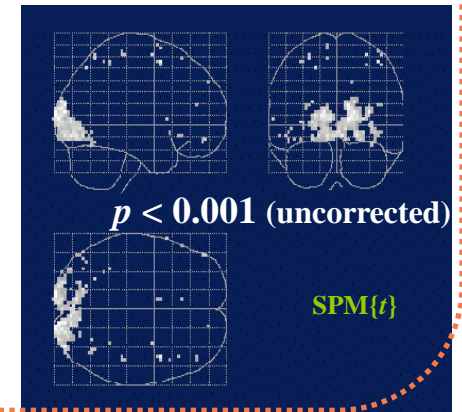
an estimate of the mixed-effects model variance

$$\hat{\sigma}^2_{\alpha} + \hat{\sigma}^2_{\epsilon} / w$$

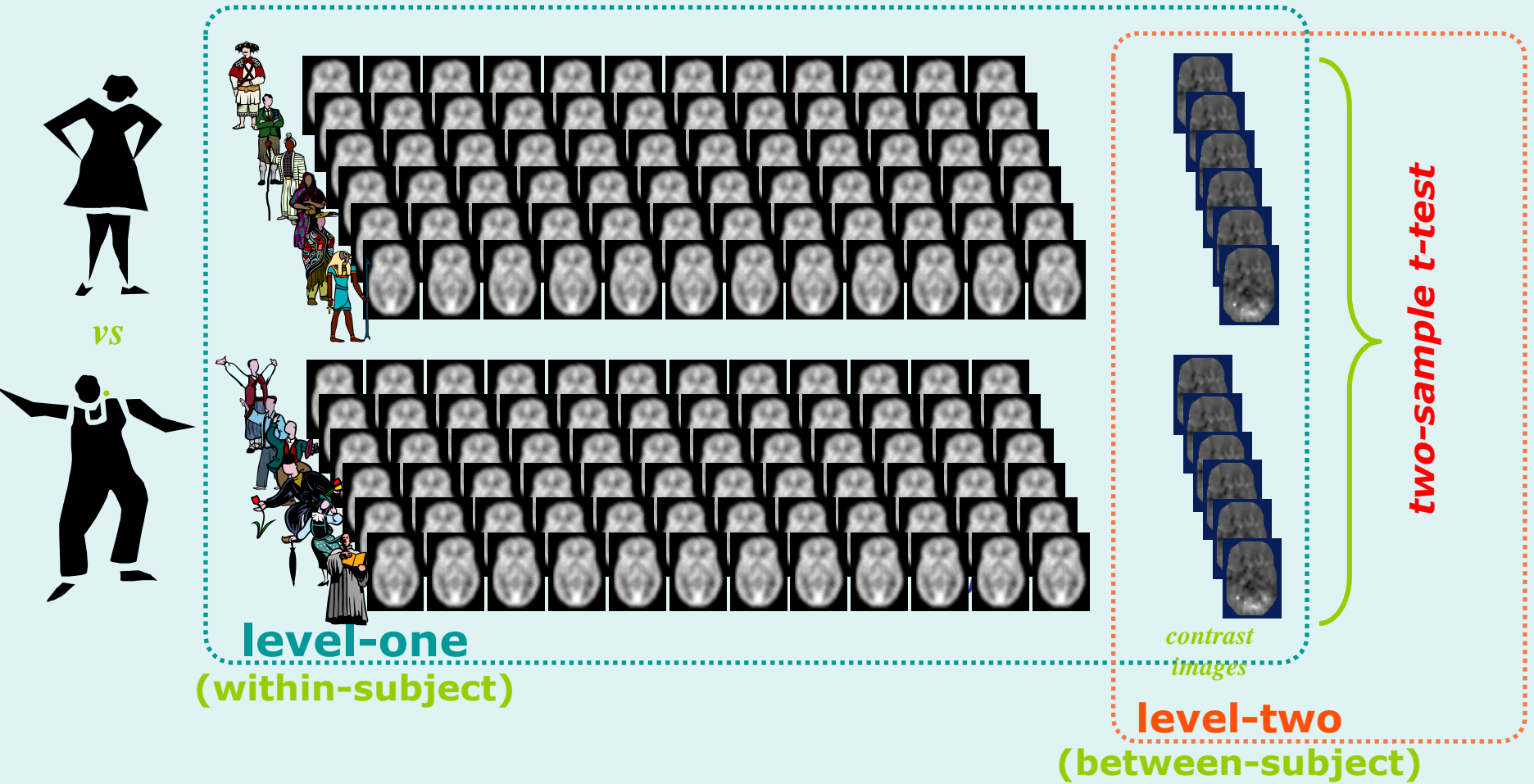
(no voxels significant at $p < 0.05$ (corrected))

$$\hat{\alpha}_{\cdot} - \text{c.f. } \hat{\sigma}^2/n = \hat{\sigma}^2_{\alpha} / n + \hat{\sigma}^2_{\epsilon} / nw$$

— c.f. |



Two stage random effects, group comparison

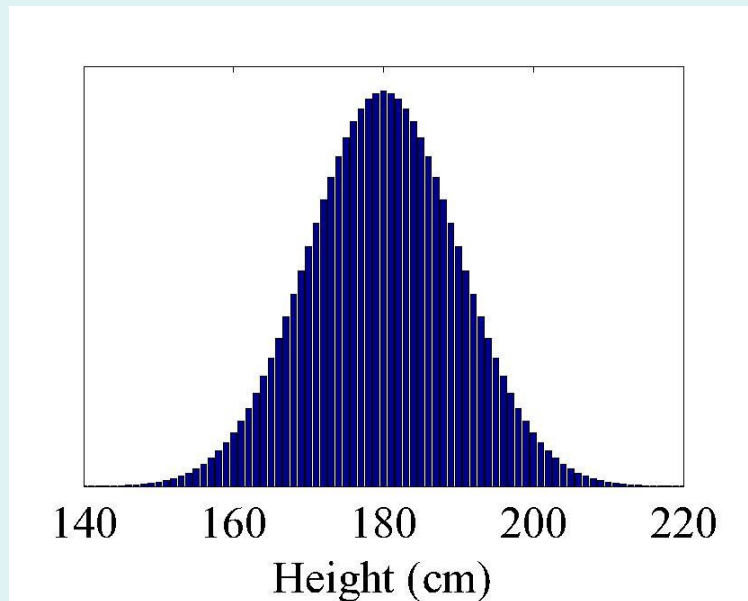


Summary

- Analyse subjects individually
 - Build within-subject models
 - Calculate contrast(s) of interest
- Use contrast images in a 2nd level (Random Effect, RFX) analysis
 - Build between-subject model
 - Calculates SPMs of interest
- Draw conclusions for the population

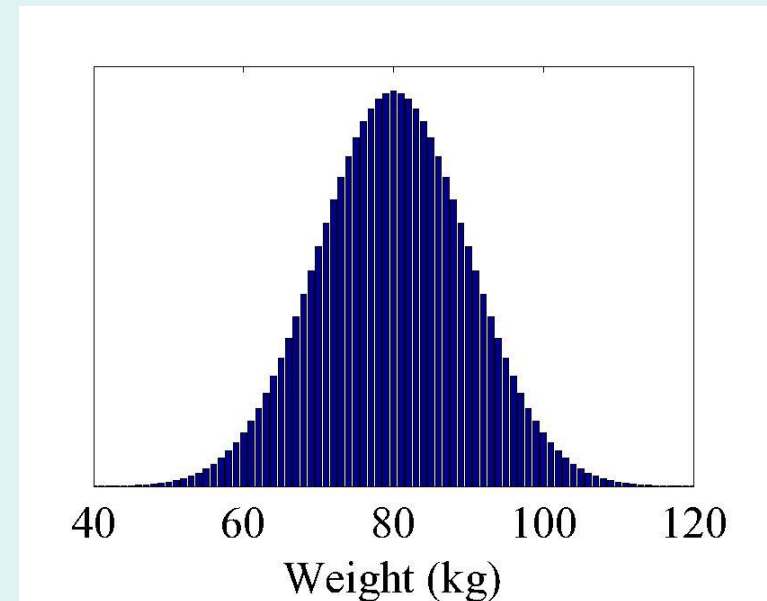
Variance-Covariance matrix

Height of Swedish men



$\mu=180\text{cm}$, $\sigma=14\text{cm}$ ($\sigma^2=200$)

Weight of Swedish men



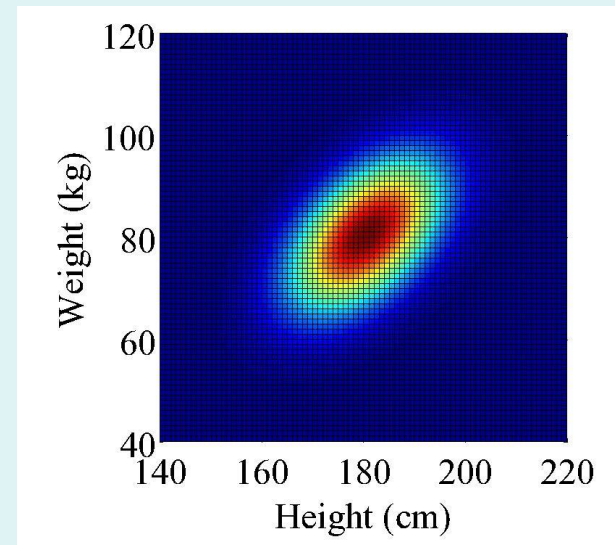
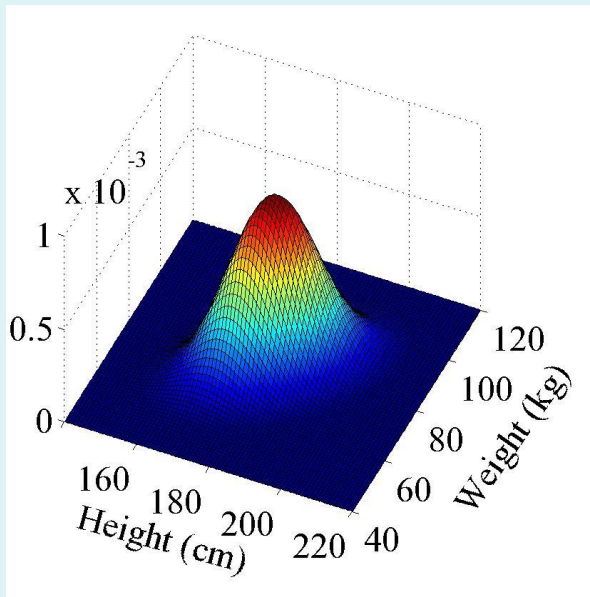
$\mu=80\text{kg}$, $\sigma=14\text{kg}$ ($\sigma^2=200$)

Each completely characterised by μ (mean) and σ^2 (variance),

i.e. we can calculate $p(l|\mu,\sigma^2)$ for any l

Variance-Covariance matrix

- Now let us view height and weight as a 2-dimensional stochastic variable ($p(l,w)$).



$$\boldsymbol{\mu} = \begin{pmatrix} 180 \\ 80 \end{pmatrix}$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} 200 & 100 \\ 100 & 200 \end{pmatrix}$$

$$p(l,w|\boldsymbol{\mu},\boldsymbol{\Sigma})$$

Sphericity

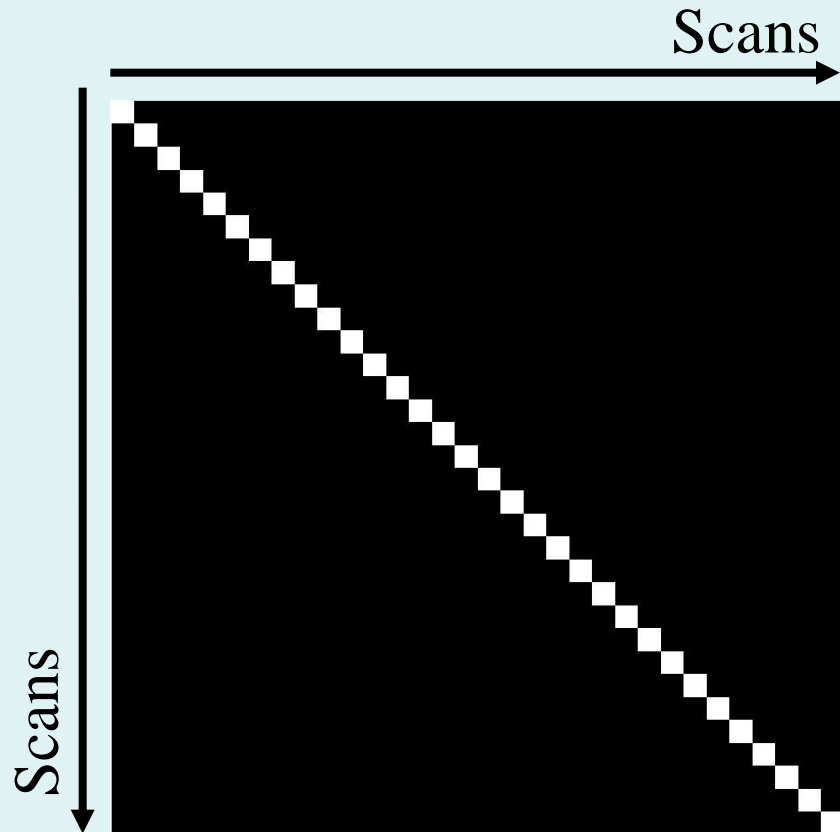
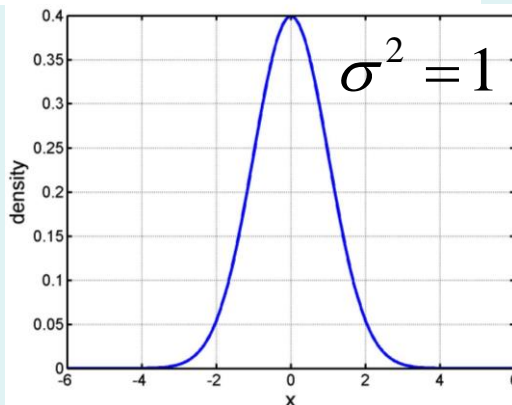
$$y = X\beta + \varepsilon$$

$$C_{\varepsilon} = \text{Cov}(\varepsilon) = E(\varepsilon\varepsilon^T)$$

„sphericity“ means:

$$\text{Cov}(\varepsilon) = \sigma^2 I$$

i.e. $\text{Var}(\varepsilon_i) = \sigma^2$

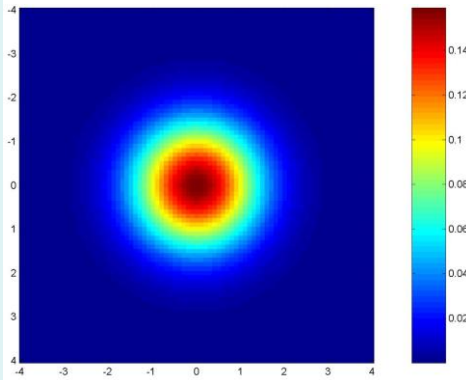


Non-sphericity

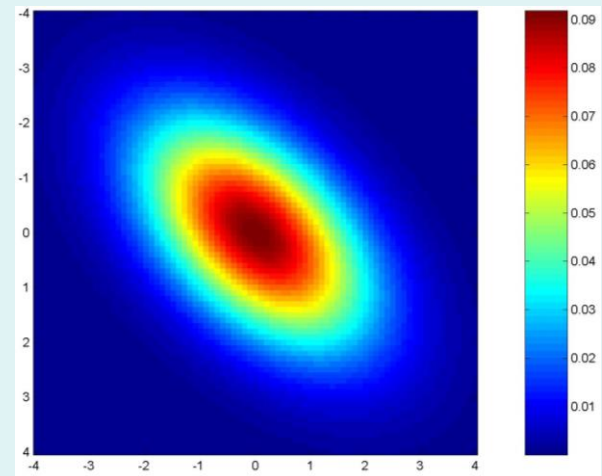
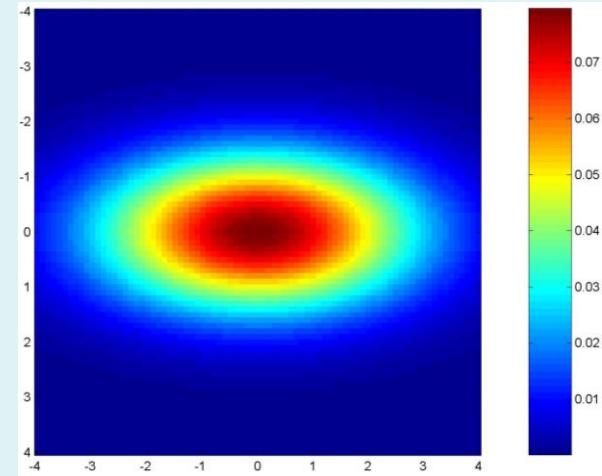
$$\text{Cov}(\varepsilon) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

non-sphericity means that the error covariance doesn't look like this:

$$\text{Cov}(\varepsilon) = \sigma^2 I$$



$$\text{Cov}(\varepsilon) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\text{Cov}(\varepsilon) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Variance quiz

Height

Weight

hours watching
telly per day

Variance quiz

Height

Weight

hours watching
telly per day

Variance quiz

Height

Weight

hours watching
telly per day

Shoe size

Variance quiz

Height

Weight

hours watching
telly per day

Shoe size

Height	Weight	# hours watching telly per day	Shoe size
Red	Red	White	Red
Red	Red	Red	White
White	Red	Red	White
Red	White	White	Red

Example I

Stimuli:

Auditory Presentation (SOA = 4 secs) of
(i) words and (ii) words spoken
backwards

e.g.
"Book"
and
"Koob"

Subjects:

(i) 12 control subjects
(ii) 11 blind subjects

Scanning:

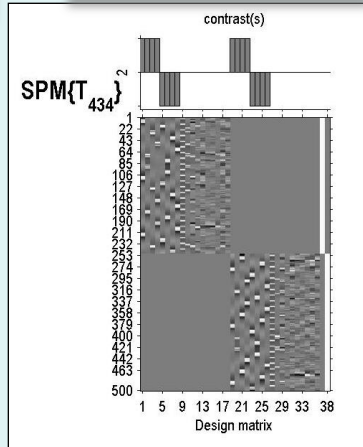
fMRI, 250 scans per
subject, block design

U. Noppeney et al.

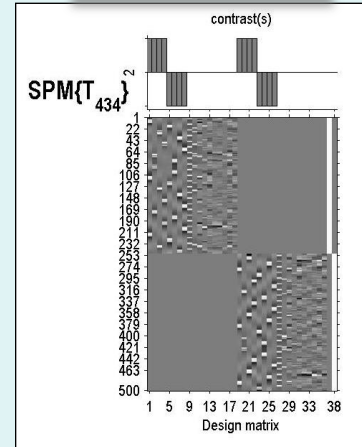
Population differences

1st level:

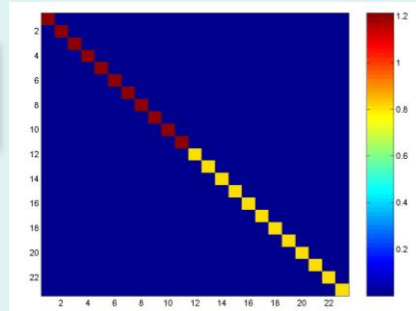
Controls



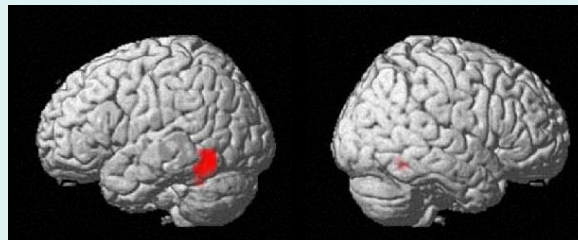
Blinds



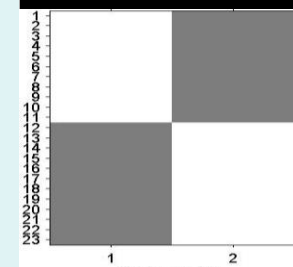
2nd level:



V



$$c^T = [1 \quad -1]$$



X

Example II

Stimuli:

Auditory Presentation (SOA = 4 secs) of words

Motion	Sound	Visual	Action
“jump”	“click”	“pink”	“turn”

Subjects:

(i) 12 control subjects

Scanning:

fMRI, 250 scans per subject, block design

Question:

What regions are affected by the semantic content of the words?

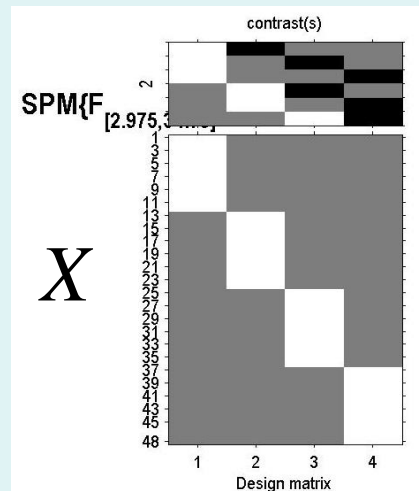
U. Noppeney et al.

SPM Notation: iid case

$$y = X \theta + \varepsilon$$

$N \times 1$ $N \times p$ $p \times 1$ $N \times 1$

- 12 subjects, 4 conditions
 - Use F-test to find differences btw conditions
- Standard Assumptions
 - Identical distribution
 - Independence
 - “Sphericity” ... but here not realistic!

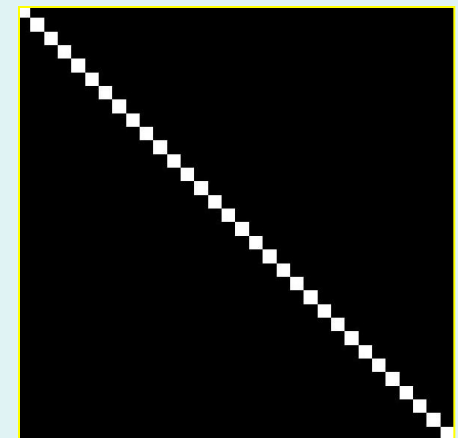


$$\text{Cor}(e) = I$$

Error covariance

N

N



Multiple Variance Components

$$y = X \theta + \varepsilon$$

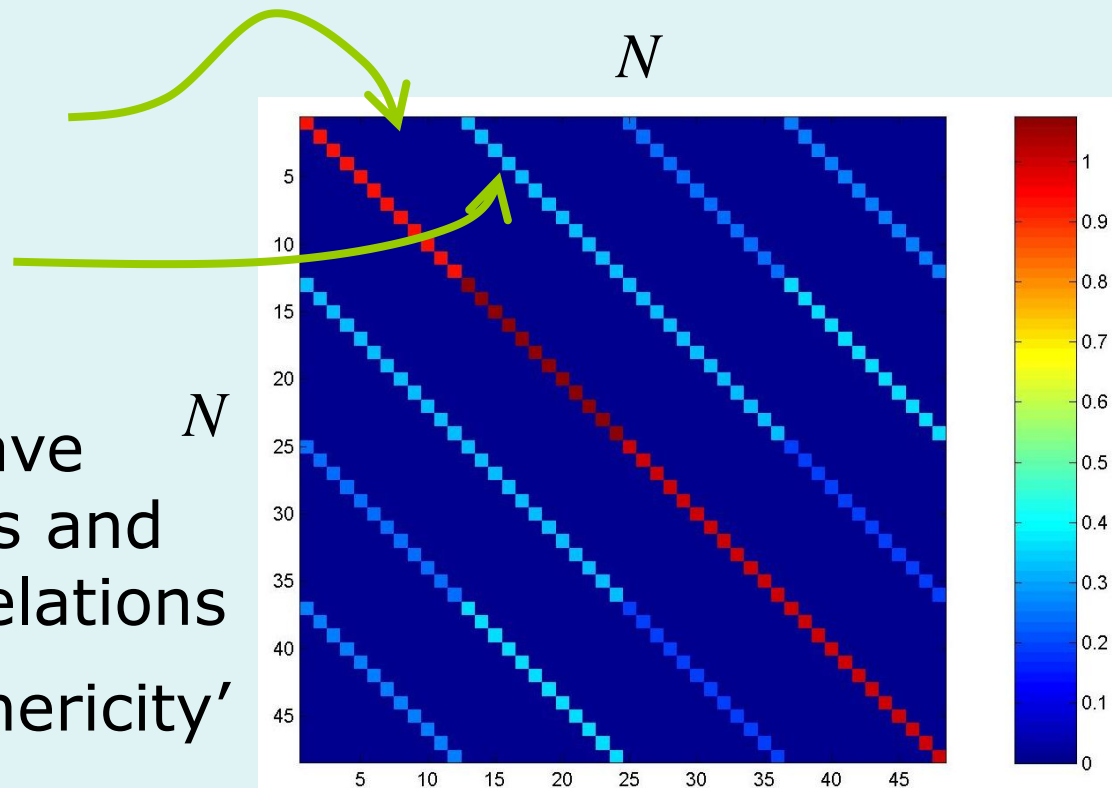
$N \times 1$ $N \times p$ $p \times 1$ $N \times 1$

$$Cor(\varepsilon) = \sum_k \lambda_k Q_k$$

- 12 subjects, 4 conditions
- Measurements btw subjects uncorrelated
- Measurements w/in subjects correlated

Error covariance

Errors can now have
different variances and
there can be correlations
Allows for 'nonsphericity'



Repeated measures Anova

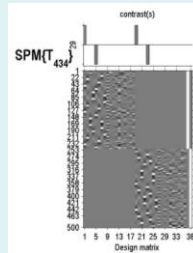
1st level:

Motion

Sound

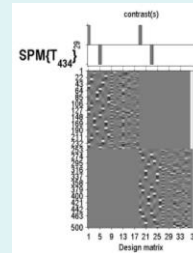
Visual

Action



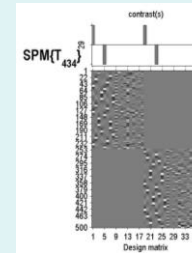
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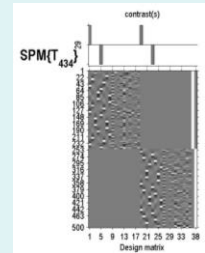
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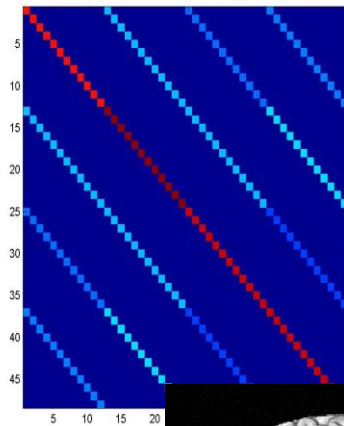


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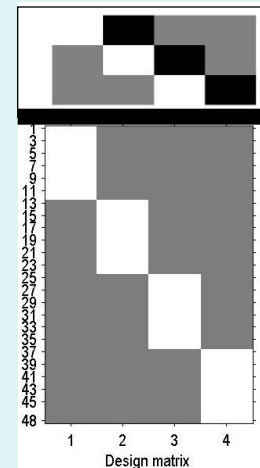
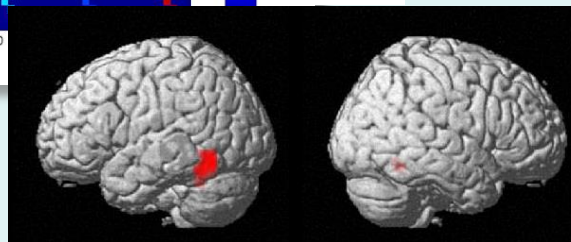
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2nd level:



V



X

$$c^T = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

