

# Introduction à la statistique médicale

## Statistical Parametric Mapping short course

### Course 5:

### Evoked response fMRI & Design efficiency

Christophe Phillips, Ir PhD  
GIGA – CRC *In Vivo* Imaging &  
GIGA – *In Silico* Medicine

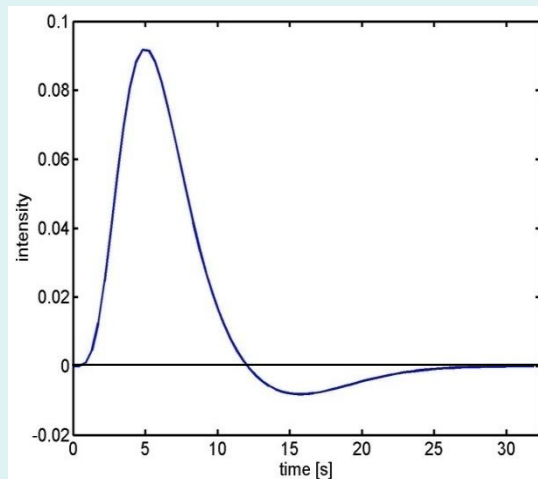
# Content

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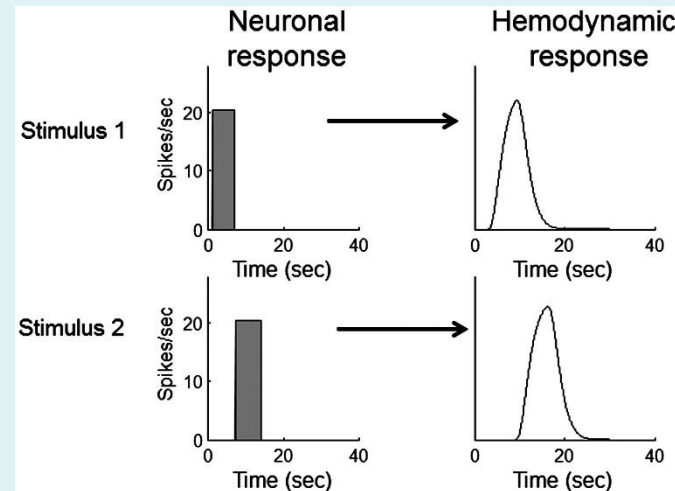
- **Evoked response models**
- Design efficiency

# BOLD response

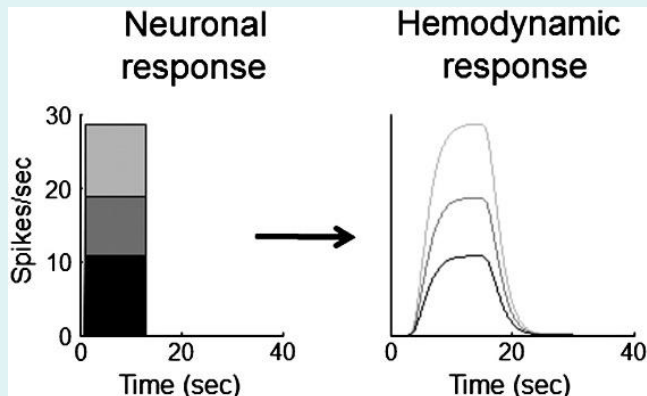
Hemodynamic response function (HRF):



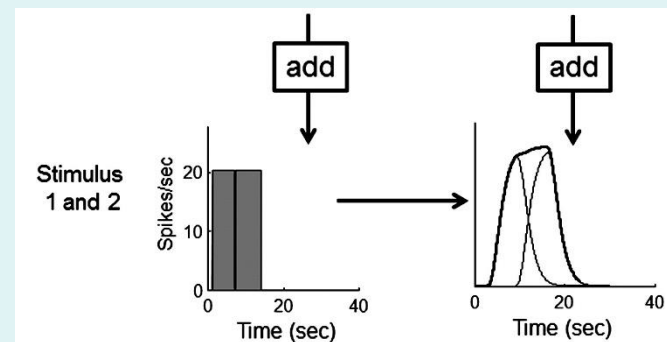
Shift invariance



Scaling

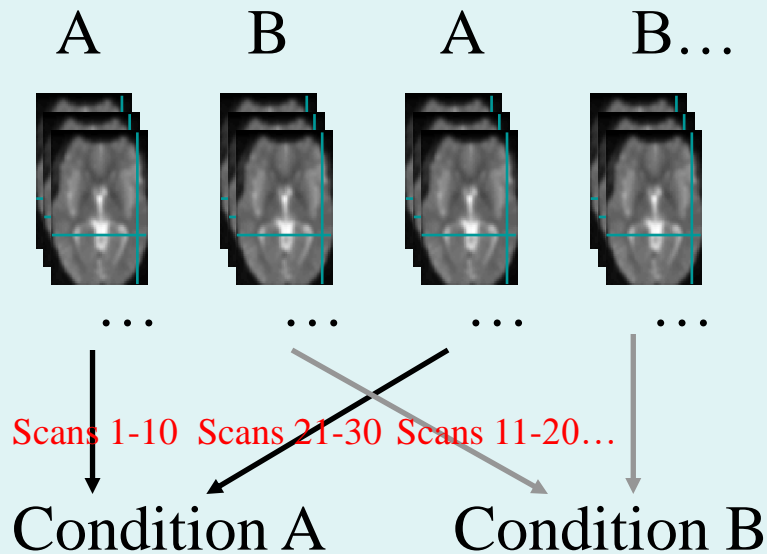


Additivity



# Epoch vs. event related design

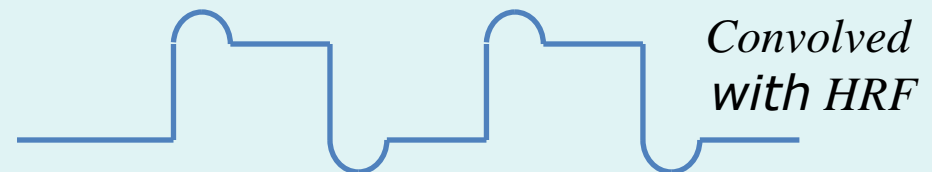
"PET **Blocked** conception"  
(scans assigned to conditions)



"fMRI **Epoch** conception"  
(scans treated as timeseries)



"fMRI **Event-related** conception"



# Advantages of Event-Related design

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- Randomised trial order  
*c.f. confounds of blocked designs*

## Blocked designs may trigger expectations and cognitive sets



...



Unpleasant (U)

Pleasant (P)

## Intermixed designs can minimise this by stimulus randomisation



...



...



...



...



...

Pleasant (P)

Unpleasant (U)

Unpleasant (U)

Pleasant (P)

Unpleasant (U)

# Advantages of Event-Related design

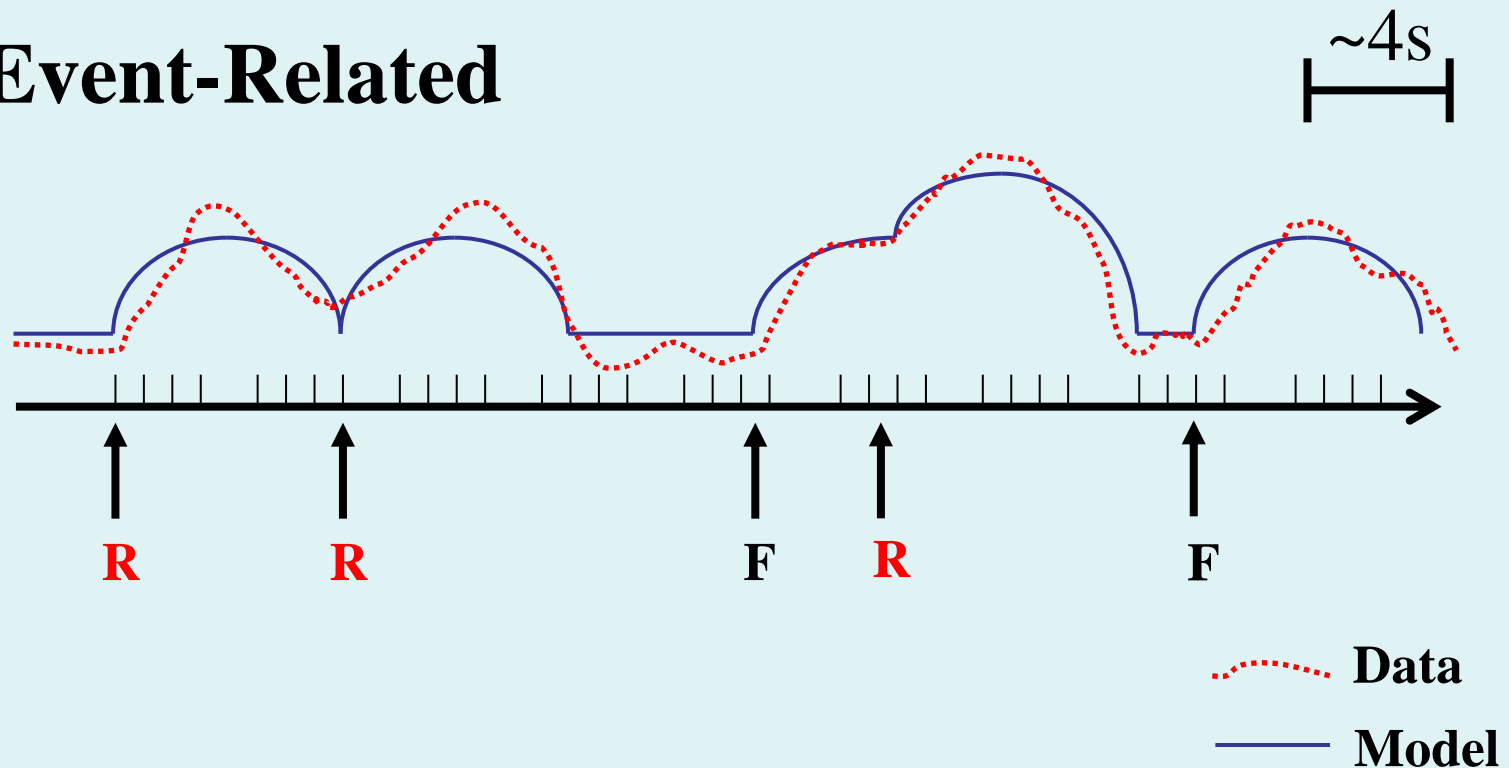
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- Randomised trial order  
*c.f. confounds of blocked designs*
- Post hoc / subjective classification of trials  
*e.g, according to subsequent memory*

**R** = Words Later Remembered

**F** = Words Later Forgotten

## Event-Related

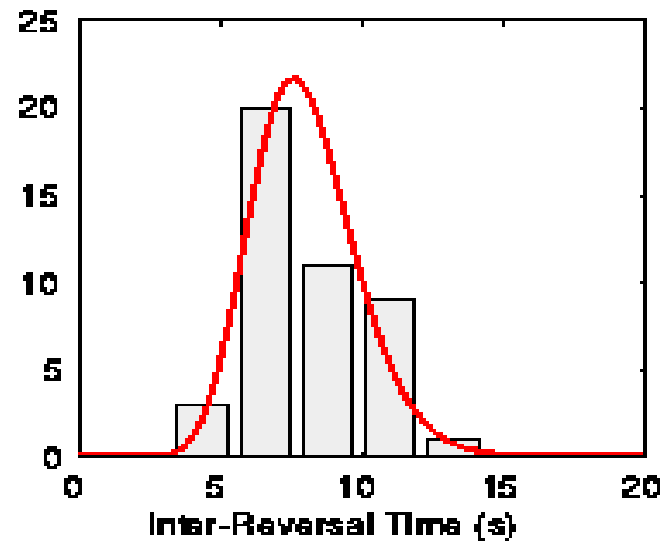
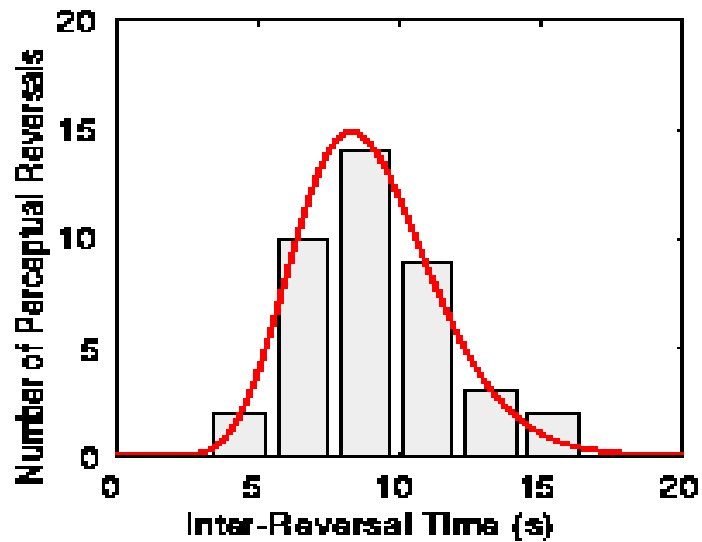




# Advantages of Event-Related design

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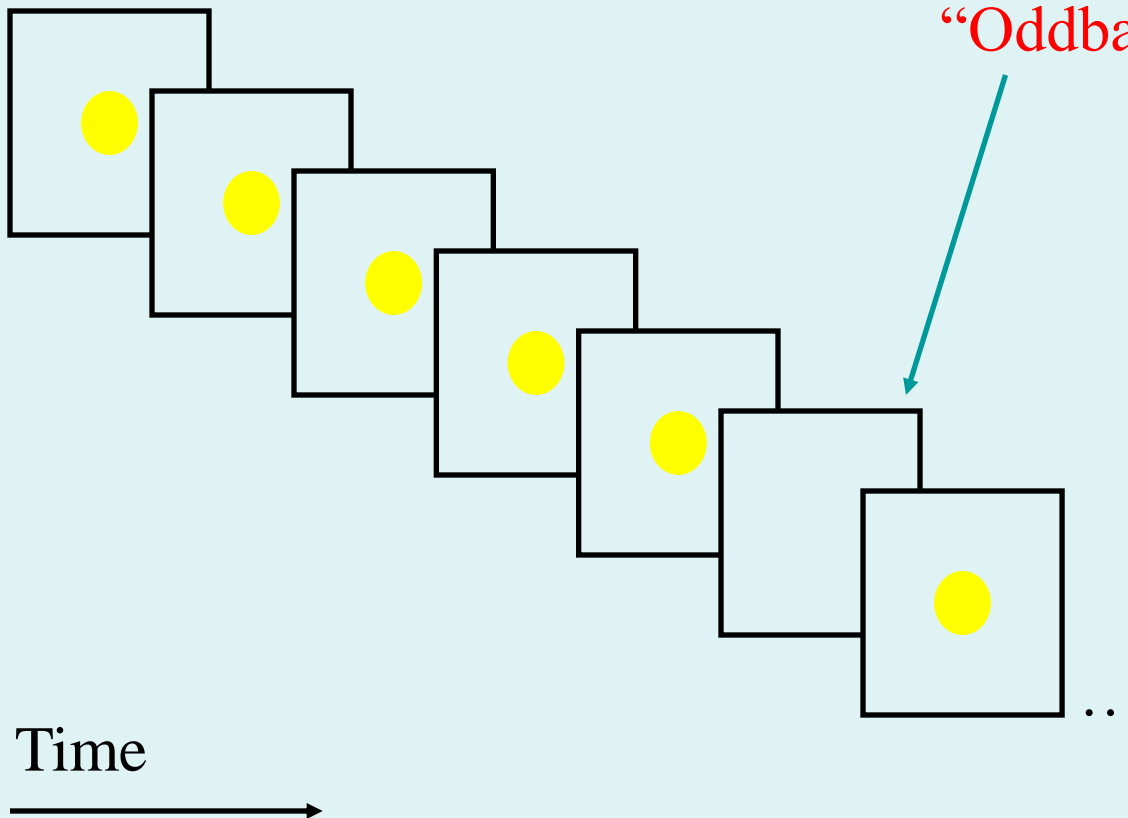
- Randomised trial order  
*c.f. confounds of blocked designs*
- Post hoc / subjective classification of trials  
*e.g, according to subsequent memory*
- Some events can only be indicated (in time)  
*e.g, spontaneous perceptual changes*



# Advantages of Event-Related design

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- Randomised trial order  
*c.f. confounds of blocked designs*
- Post hoc / subjective classification of trials  
*e.g, according to subsequent memory*
- Some events can only be indicated (in time)  
*e.g, spontaneous perceptual changes*
- Some trials cannot be blocked  
*e.g, “oddball” designs*



# Advantages of Event-Related design

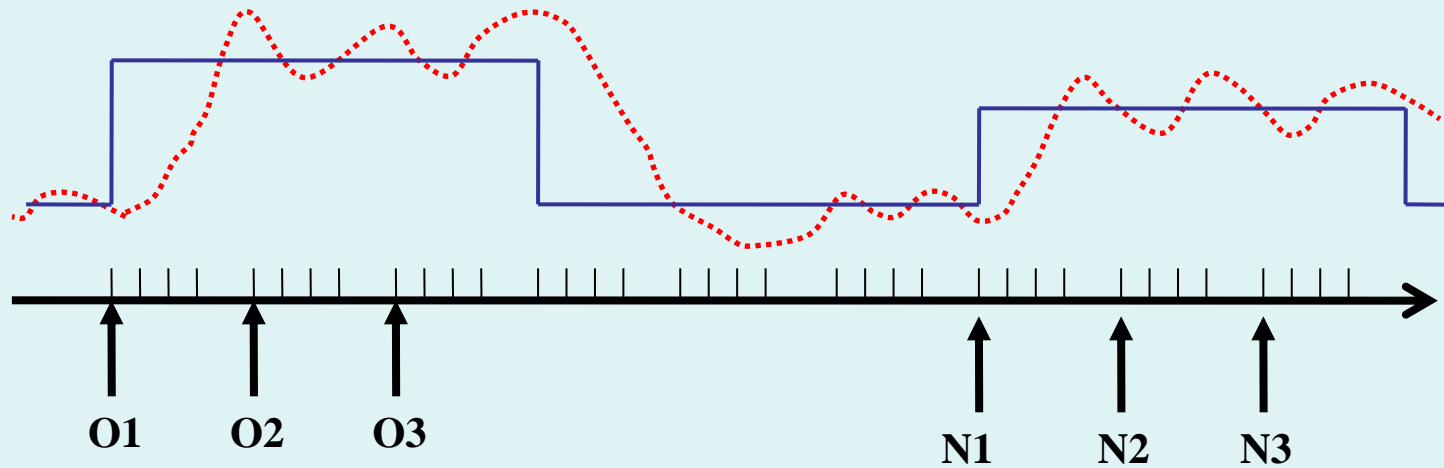
---

- Randomised trial order  
*c.f. confounds of blocked designs*
- Post hoc / subjective classification of trials  
*e.g, according to subsequent memory*
- Some events can only be indicated (in time)  
*e.g, spontaneous perceptual changes*
- Some trials cannot be blocked  
*e.g, “oddball” designs*
- More accurate models even for blocked designs?  
*e.g, “state-item” interactions*

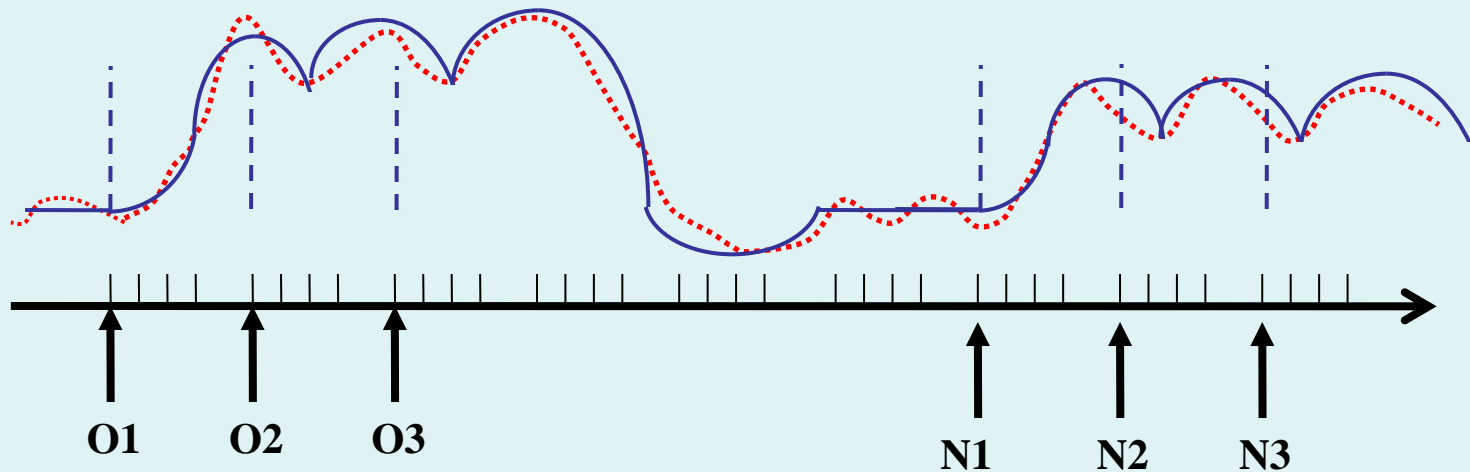
# Blocked Design

..... Data  
—— Model

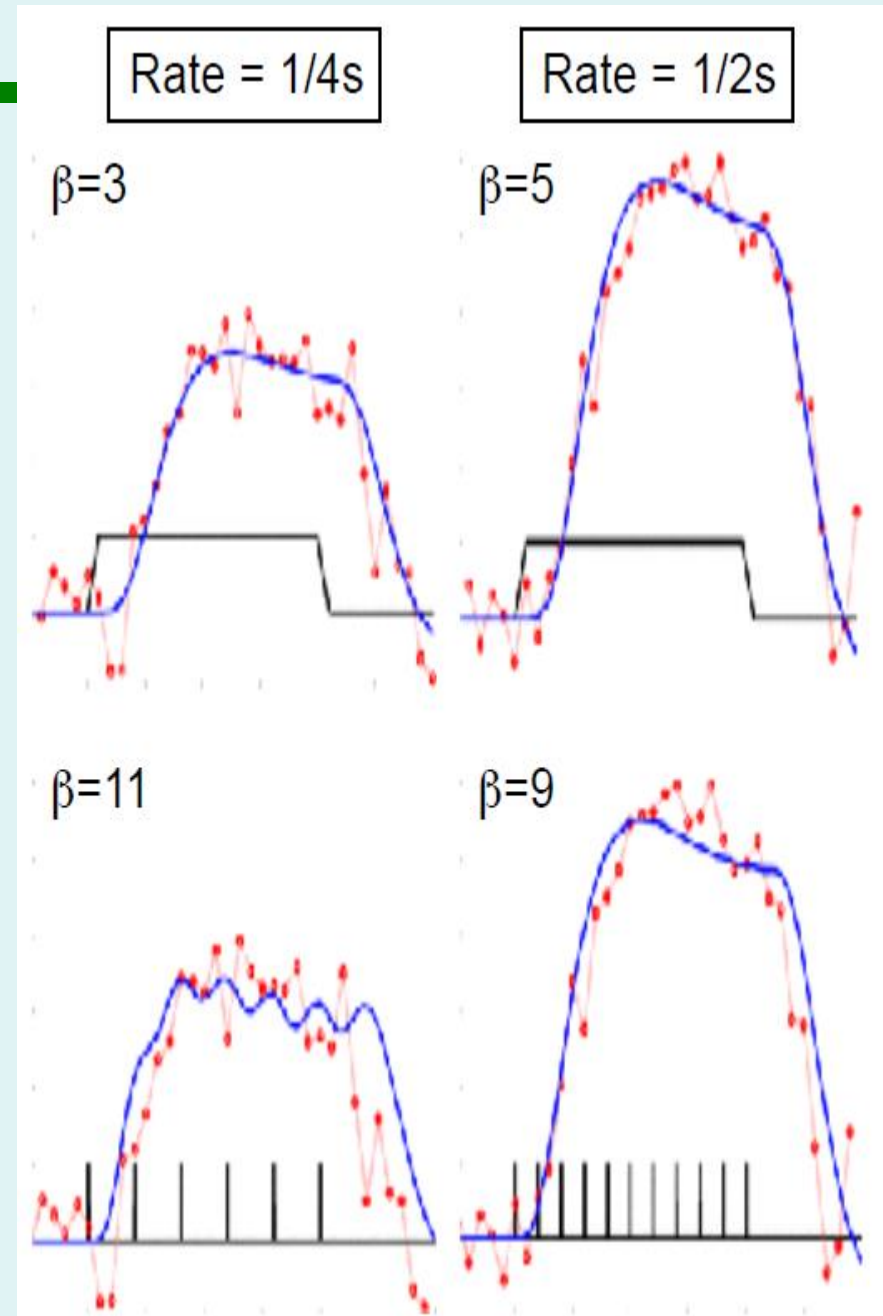
## “Epoch” model



## “Event” model



- Blocks of trials can be modeled as boxcars or runs of events
- BUT: interpretation of the parameter estimates may differ
- Consider an experiment presenting words at different rates in different blocks:
  - ▶ An “epoch” model will estimate parameter that increases with rate, because the parameter reflects response per block
  - ▶ An “event” model may estimate parameter that decreases with rate, because the parameter reflects response per word



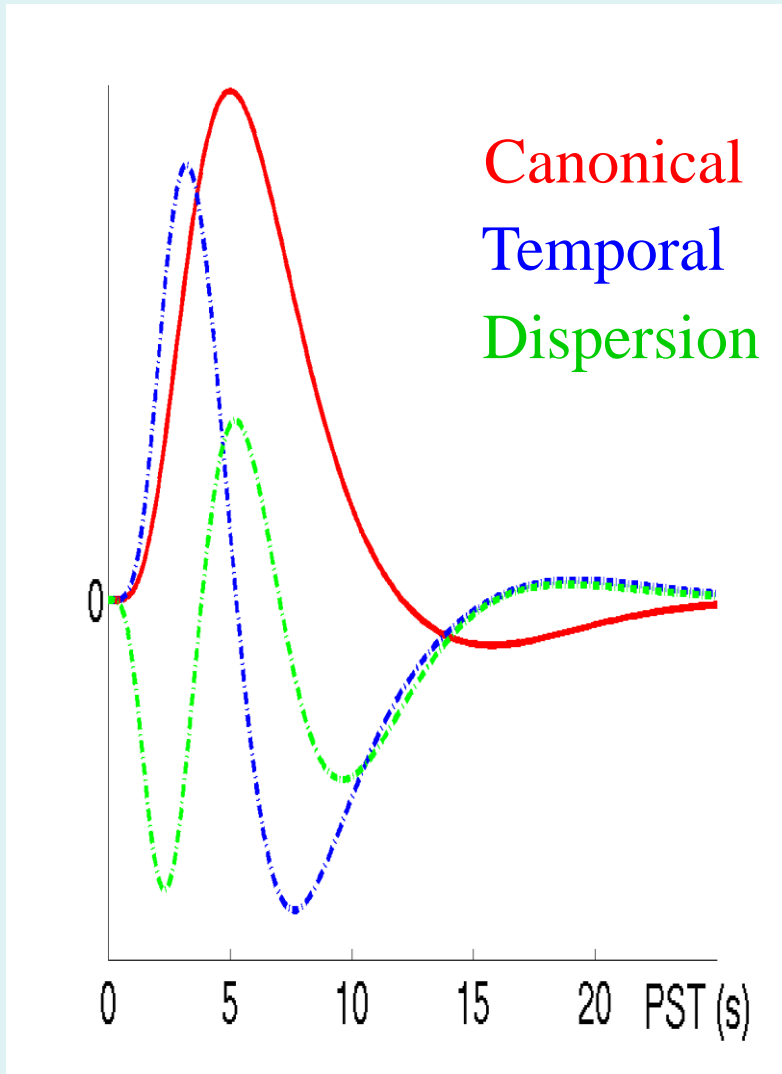
# Disadvantages of ER designs

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- Less efficient for detecting effects than are blocked designs (*see later...*)
- Some psychological processes may be better blocked (e.g. task-switching, attentional instructions)



# Temporal Basis Functions



- Informed Basis Set

(Friston et al. 1998)

- Canonical HRF (2 gamma functions)

*plus* Multivariate Taylor expansion in:

time (*Temporal Derivative*)

width (*Dispersion Derivative*)

- “Magnitude” inferences via t-test on canonical parameters (providing canonical is a good fit...more later)
- “Latency” inferences via tests on *ratio* of derivative : canonical parameters (more later...)

# Temporal Basis Functions

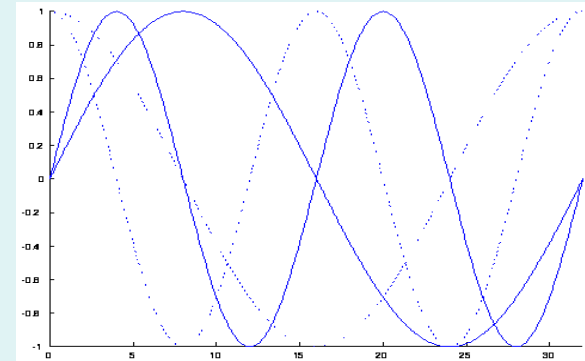
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- **Fourier Set**

Windowed sines & cosines

Any shape (up to frequency limit)

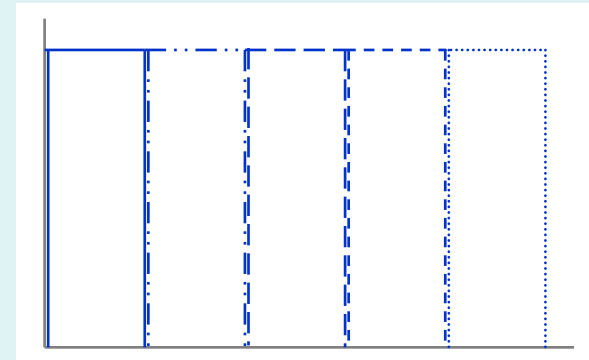
Inference via F-test



# Temporal Basis Functions

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- Finite Impulse Response (FIR)
  - Mini timebins (selective averaging)
  - Any shape (up to bin-width)
  - Inference via F-test



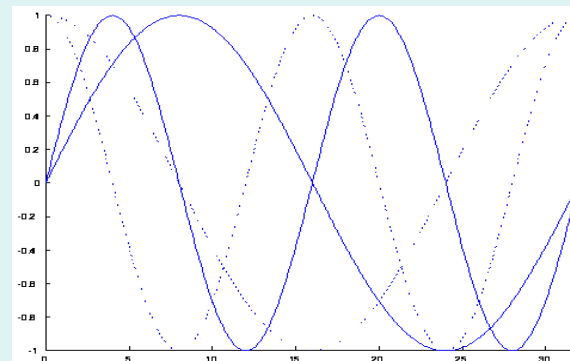
# Temporal Basis Functions

- **Fourier Set**

  - Windowed sines & cosines

  - Any shape (up to frequency limit)

  - Inference via F-test

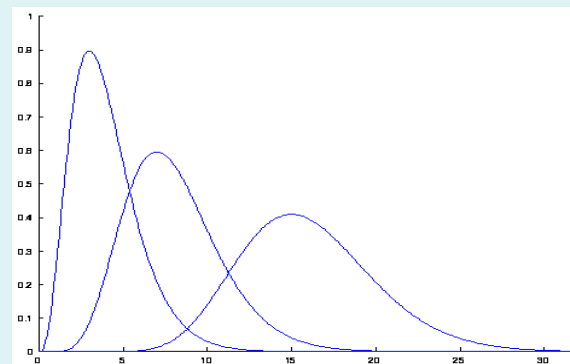


- **Gamma Functions**

  - Bounded, asymmetrical (like BOLD)

  - Set of different lags

  - Inference via F-test



# Temporal Basis Functions

- **Fourier Set**

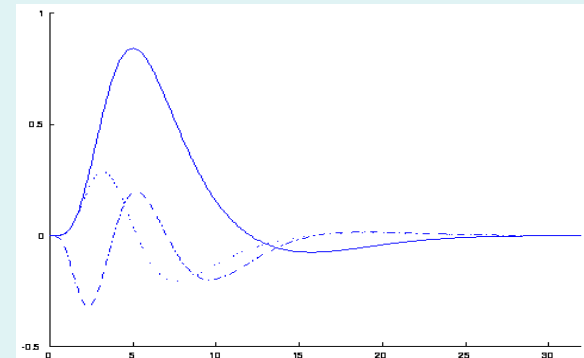
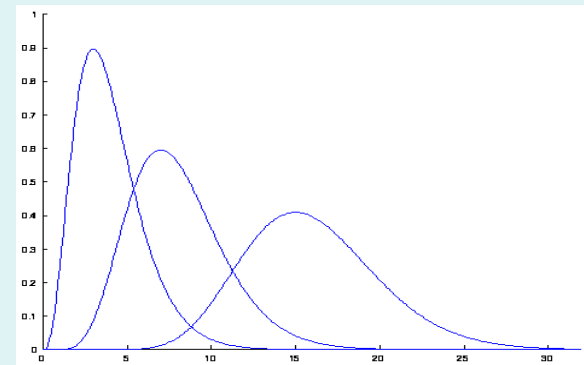
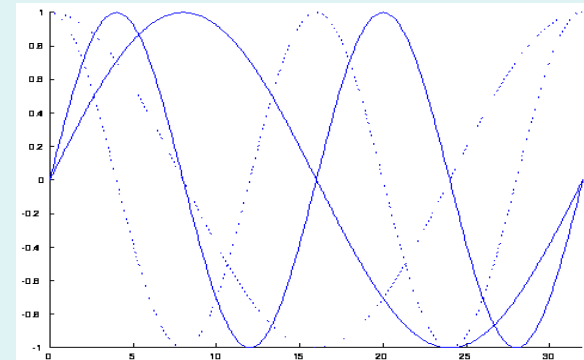
- Windowed sines & cosines
  - Any shape (up to frequency limit)
  - Inference via F-test

- **Gamma Functions**

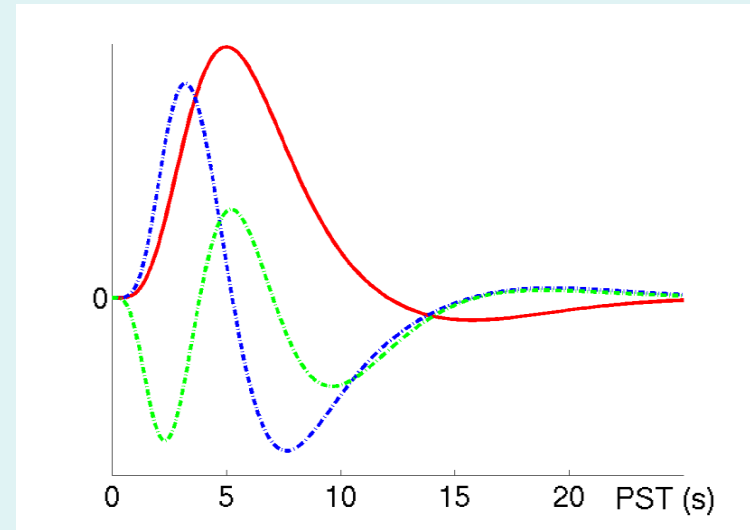
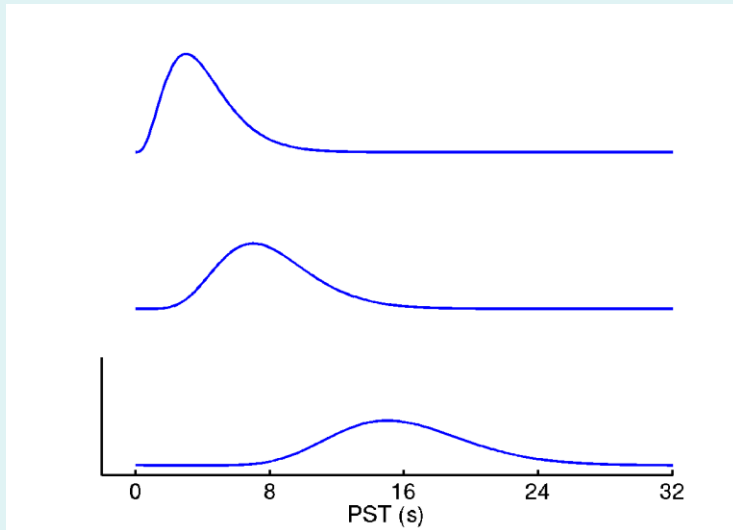
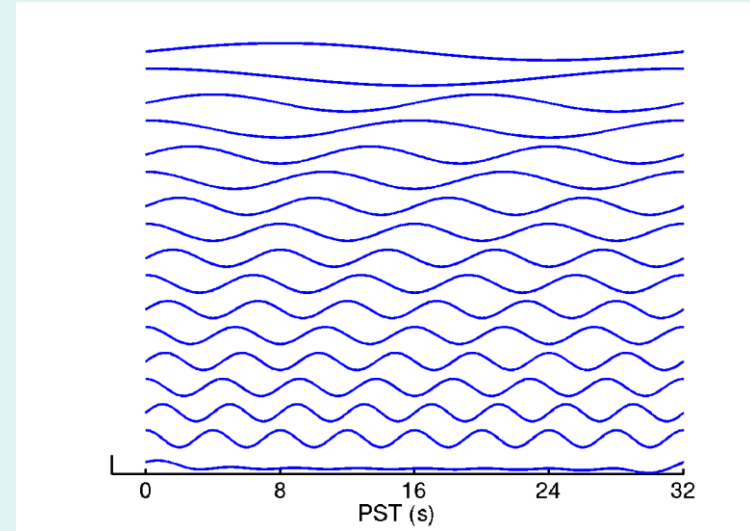
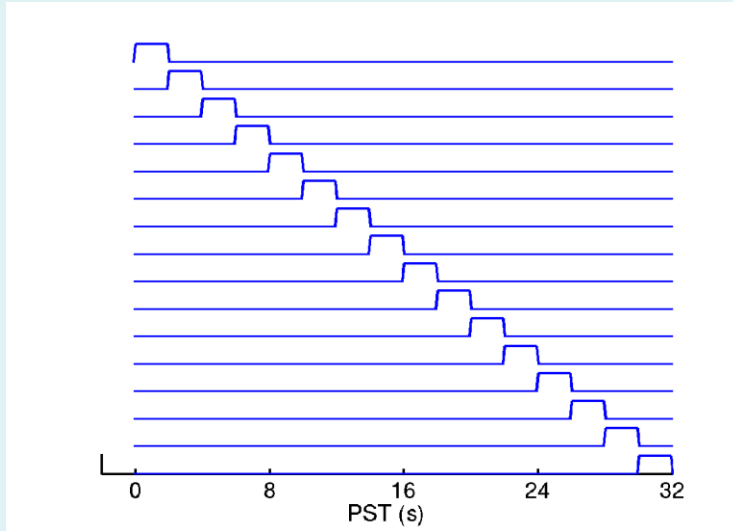
- Bounded, asymmetrical (like BOLD)
  - Set of different lags
  - Inference via F-test

- **Informed Basis Set**

- Best guess of canonical BOLD response
  - Variability captured by Taylor expansion
  - “Magnitude” inferences via t-test...?

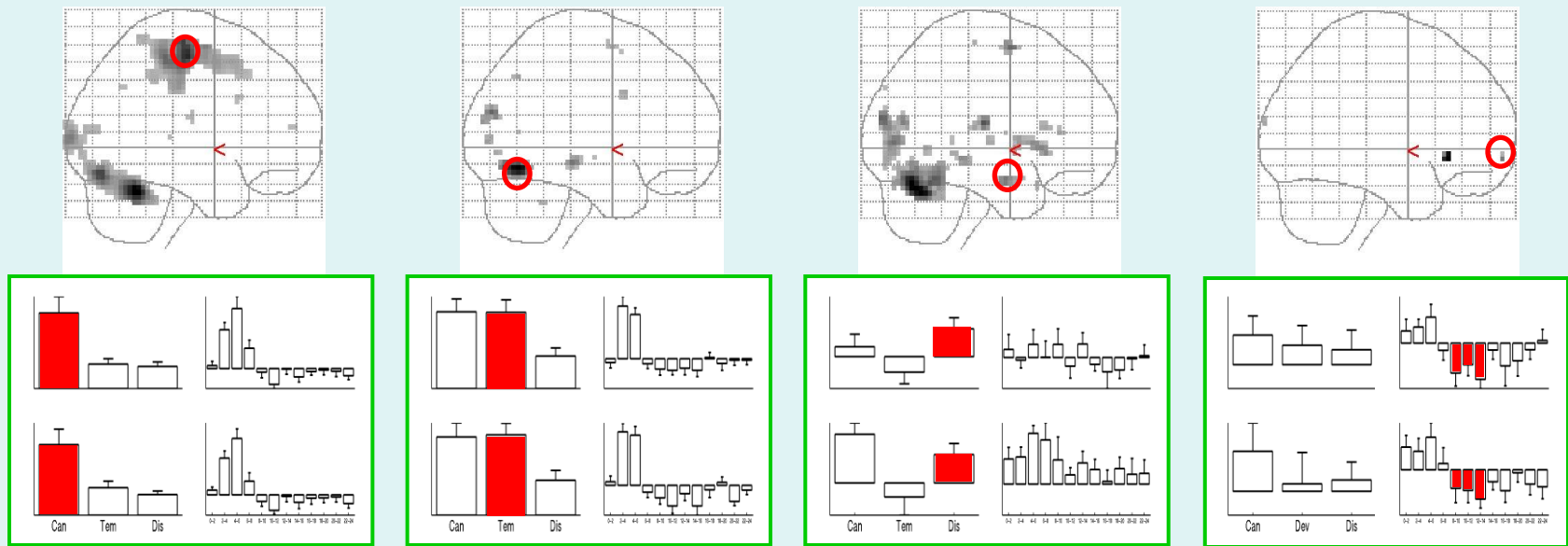


# Temporal Basis Functions



# Temporal Basis Functions, which one(s)?

In this example (rapid motor response to faces, *Henson et al, 2001*)...



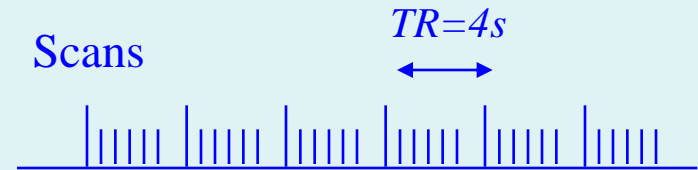
**Canonical + Temporal + Dispersion + FIR**

...canonical + temporal + dispersion derivatives appear sufficient  
...may not be for more complex trials (eg stimulus-delay-response)  
...but then such trials better modelled with separate neural components (ie activity no longer delta function) + constrained HRF (Zarahn, 1999)

# Timing Issues

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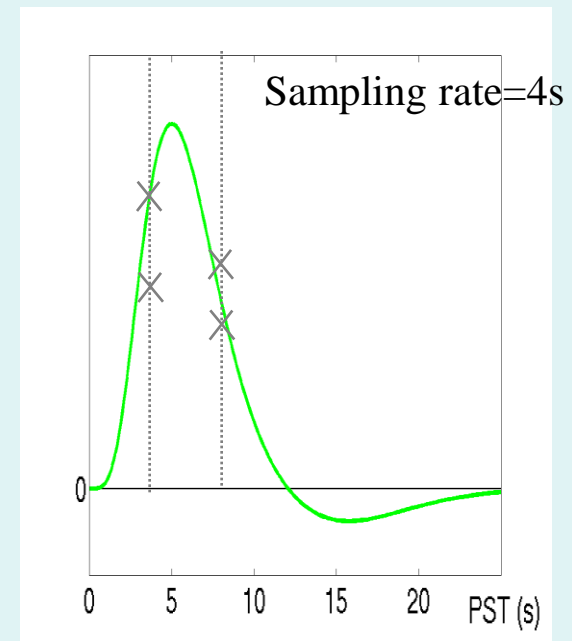
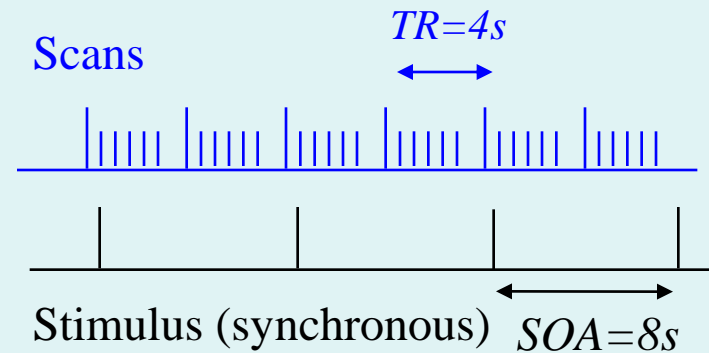
- Typical TR for 48 slice EPI at 3mm spacing is  $\sim 4s$





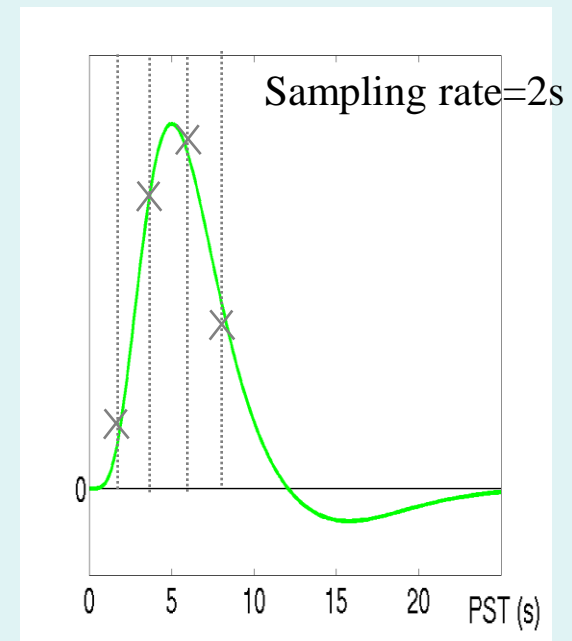
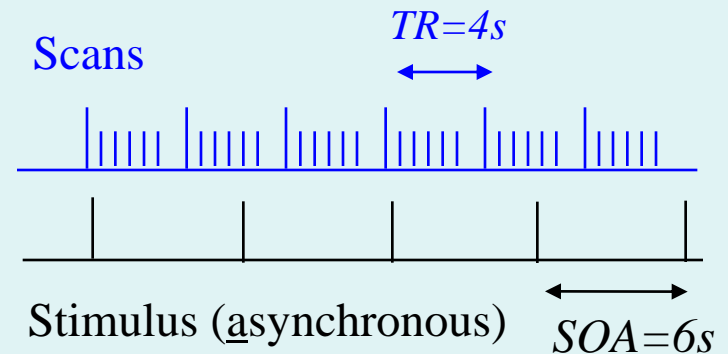
# Timing Issues

- Typical TR for 48 slice EPI at 3mm spacing is  $\sim 4s$
- Sampling at  $[0,4,8,12\dots]$  post-stimulus may miss peak signal



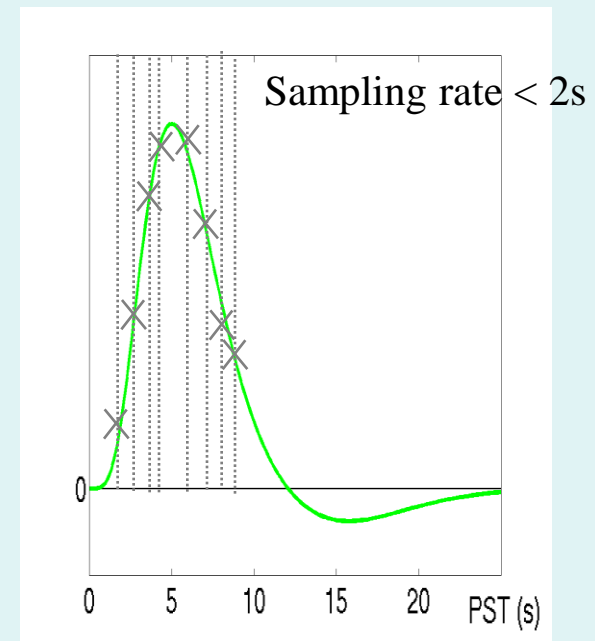
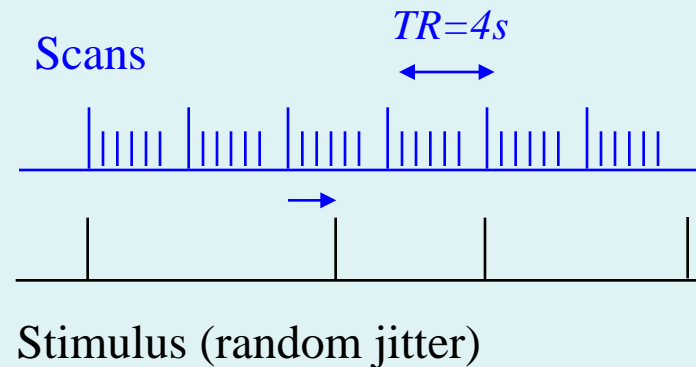
# Timing Issues

- Typical TR for 48 slice EPI at 3mm spacing is  $\sim 4s$
- Sampling at  $[0,4,8,12\dots]$  post-stimulus may miss peak signal
- Higher effective sampling by:
  1. Asynchrony, e.g.  $SOA=1.5TR$



# Timing Issues

- Typical TR for 48 slice EPI at 3mm spacing is  $\sim 4s$
- Sampling at  $[0,4,8,12\dots]$  post-stimulus may miss peak signal
- Higher effective sampling by:
  1. Asynchrony, e.g.  
 $SOA = 1.5TR$
  2. Random Jitter, e.g.  
 $SOA = (2 \pm 0.5)TR$



# BOLD Response Latency (Linear)

- Assume the real response,  $r(t)$ , is a scaled (by  $\alpha$ ) version of the canonical,  $f(t)$ , but delayed by a small amount  $dt$ :

$$r(t) = \alpha f(t+dt) \sim \alpha f(t) + \alpha f'(t) dt \quad \text{1st-order Taylor}$$

- If the fitted response,  $R(t)$ , is modelled by the canonical + temporal derivative:

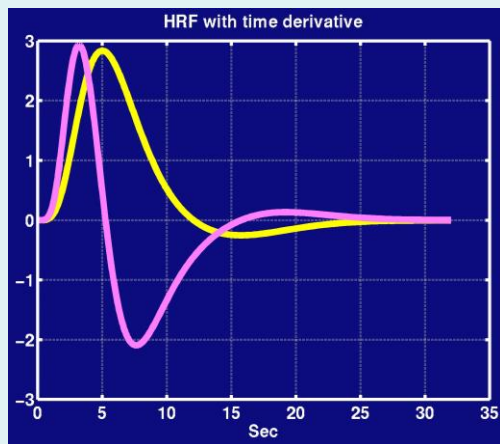
$$R(t) = \beta_1 f(t) + \beta_2 f'(t) \quad \text{GLM fit}$$

- Then canonical and derivative parameter estimates,  $\beta_1$  and  $\beta_2$ , are such that:

$$\alpha = \beta_1, \quad dt = \beta_2 / \beta_1$$

- *i.e. latency can be approximated by the ratio of derivative-to-canonical parameter estimates (within limits of first-order approximation, +/- 1s)*

# BOLD Response Latency: example



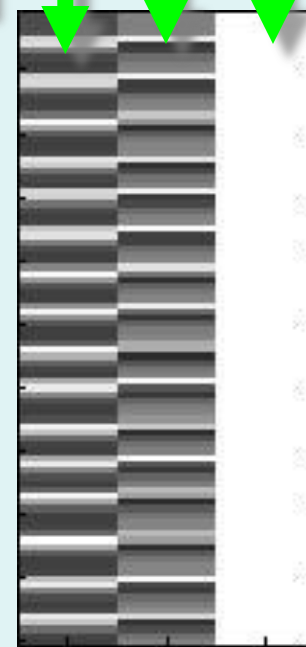
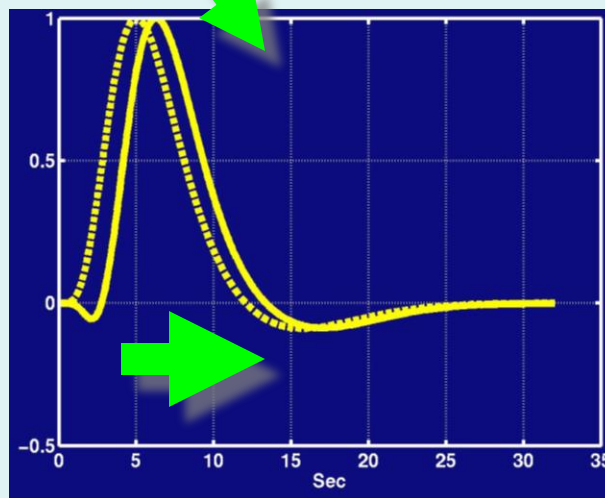
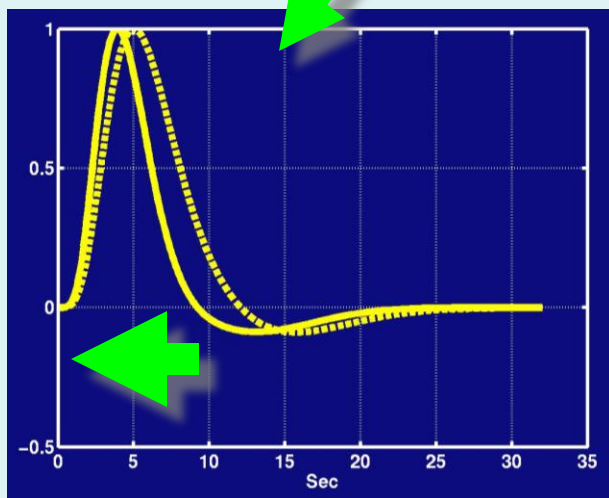
constant

derivative

HRF

Positive

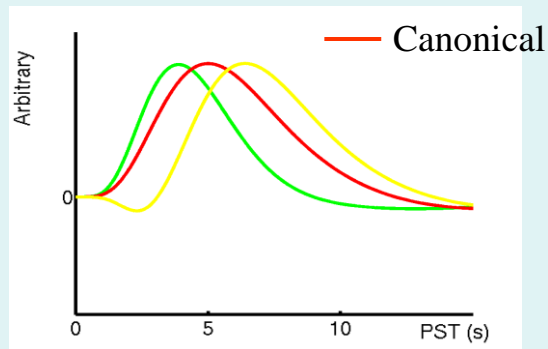
Negative



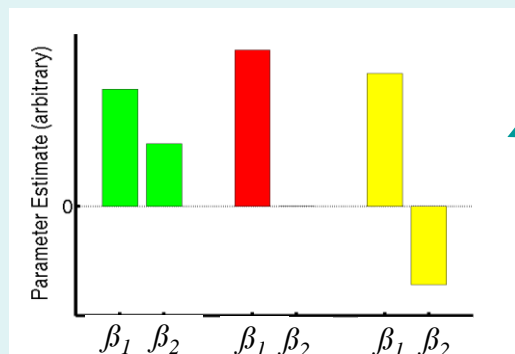
**Designmatrix**  
16 events, SOA ~18s,  
TR 3s

# BOLD Response Latency (Linear)

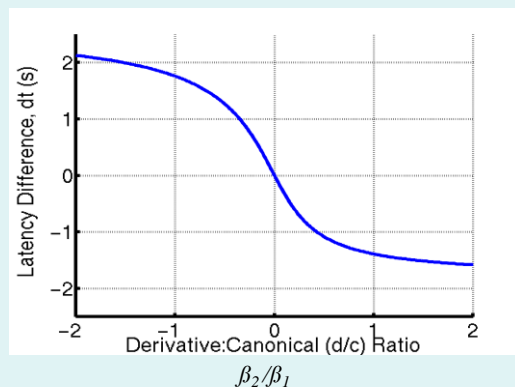
Delayed Responses  
(green/yellow)



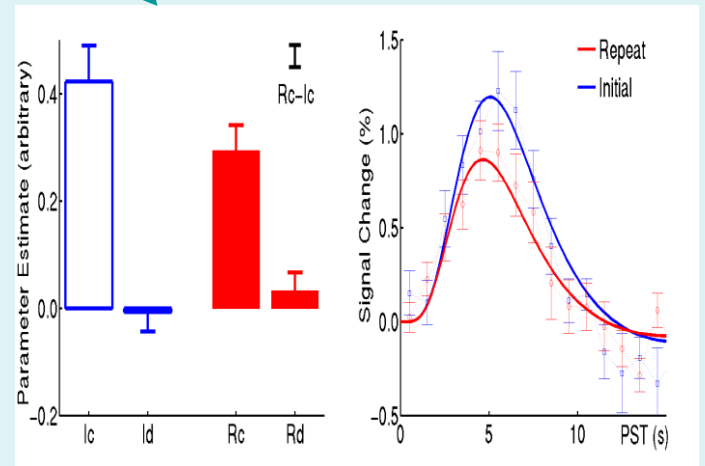
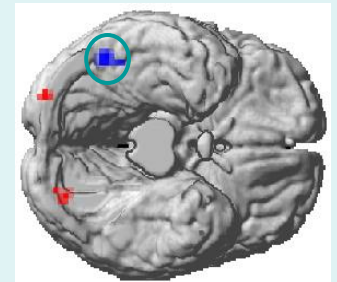
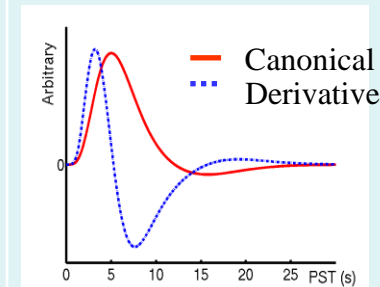
Parameter Estimates



Actual latency,  $dt$ , vs.  $\beta_2/\beta_1$



## Basis Functions



*Face repetition reduces latency as well as magnitude of fusiform response*

# Neural Response Latency

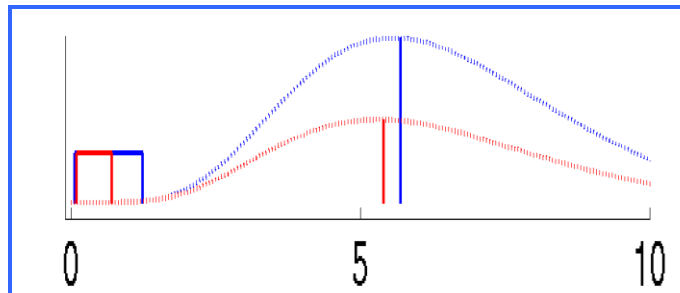
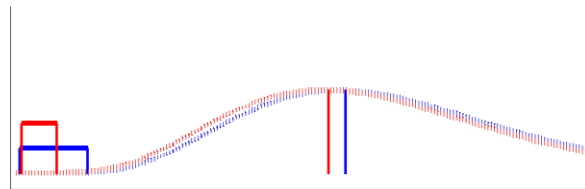
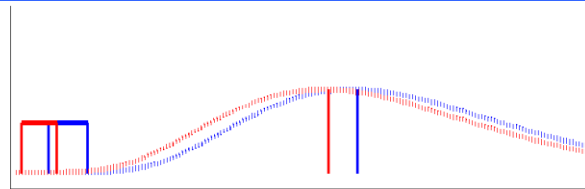
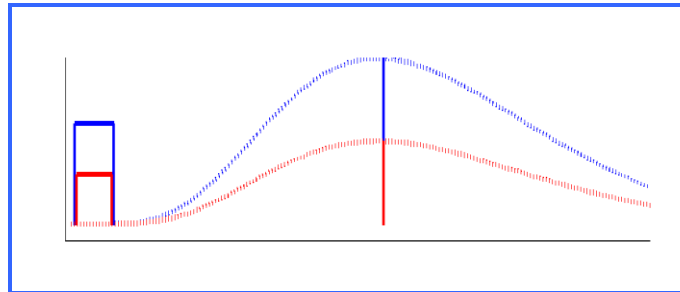
*Neural*

A. Decreased

B. Advanced

C. Shortened  
(same  
integrated)

D. Shortened  
(same  
maximum)



*BOLD*

A. Smaller Peak

B. Earlier Onset

C. Earlier Peak

D. Smaller Peak  
and earlier Peak

# Content

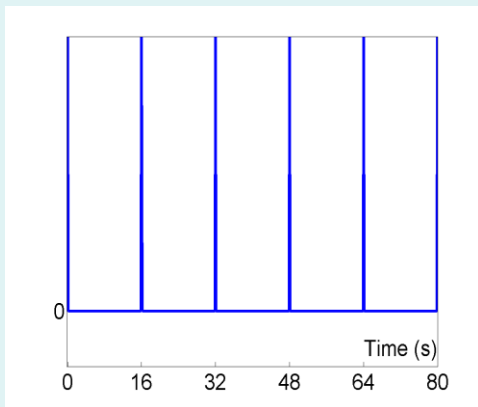
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- Evoked response models
- **Design efficiency**

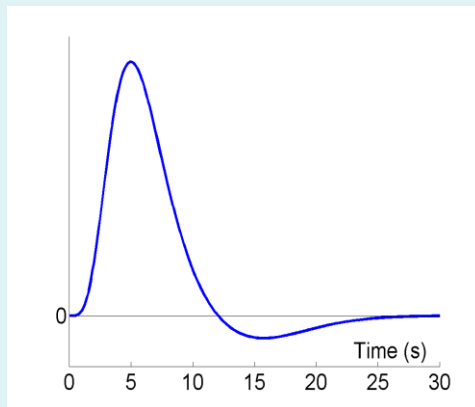


# Fixed SOA = 16s

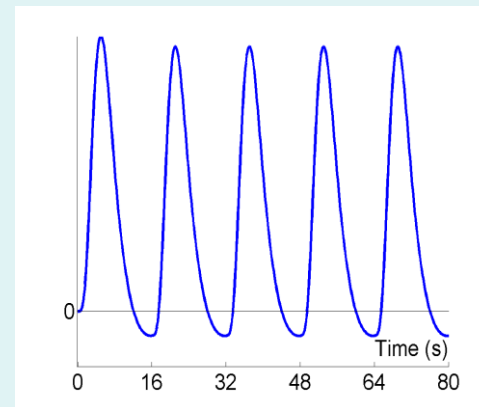
Stimulus (“Neural”)



HRF



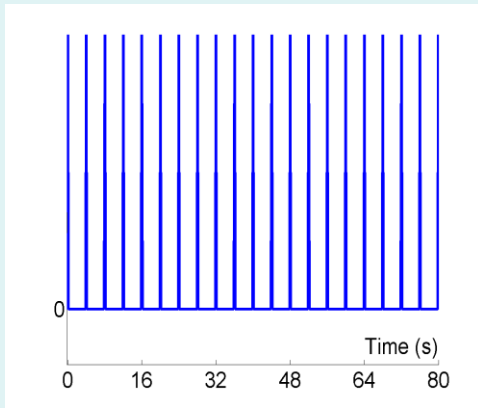
Predicted Data



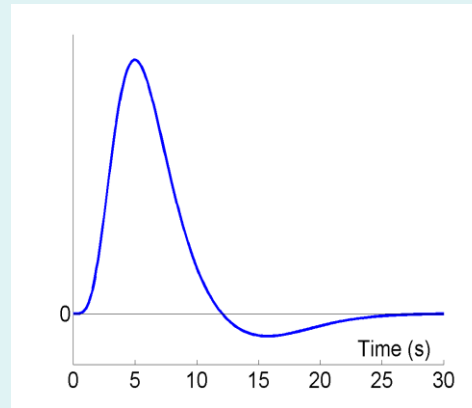
*Not particularly efficient...*

# Fixed SOA = 4s

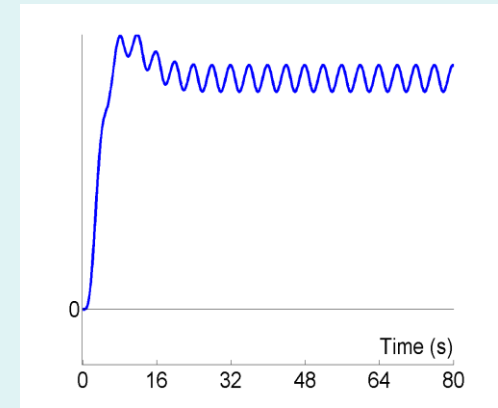
Stimulus (“Neural”)



HRF



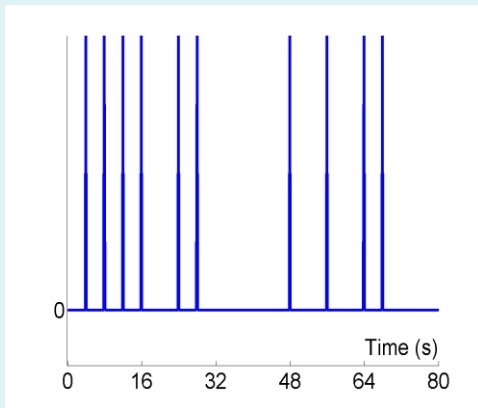
Predicted Data



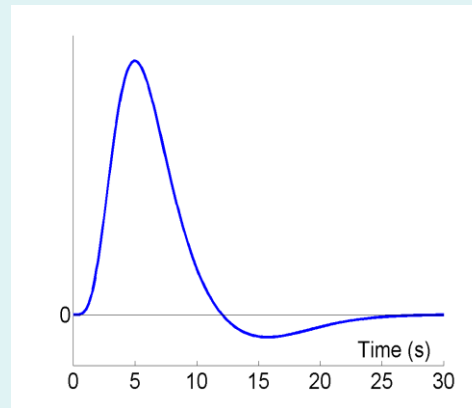
*Very Inefficient...*

# Randomised, $SOA_{\min} = 4s$

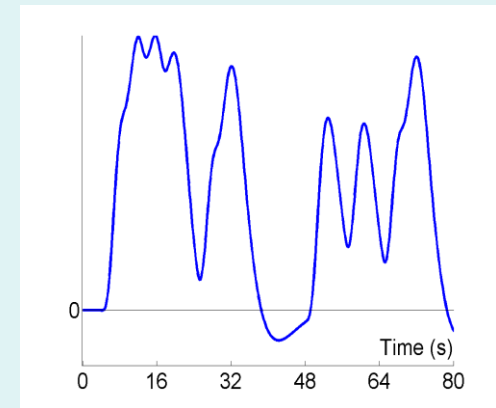
Stimulus (“Neural”)



HRF



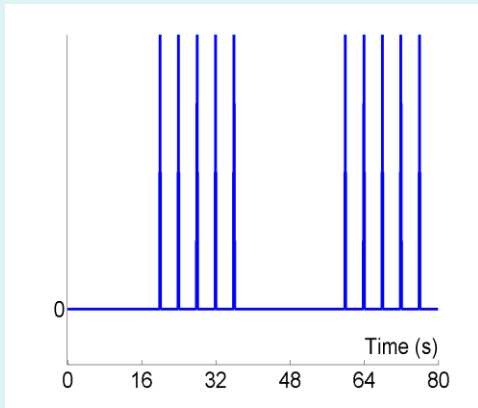
Predicted Data



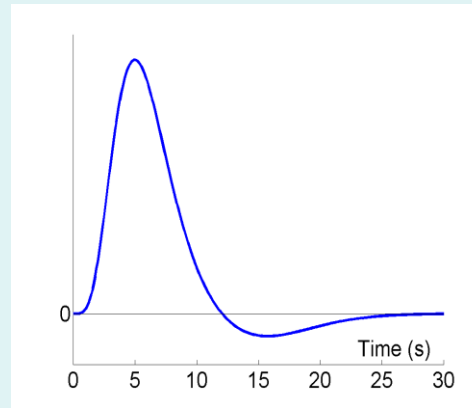
*More Efficient...*

# Blocked, $SOA_{\min} = 4s$

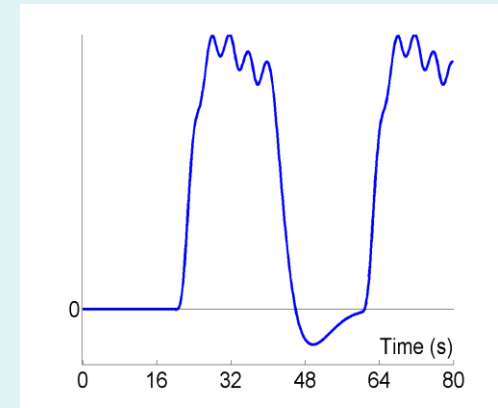
Stimulus (“Neural”)



HRF



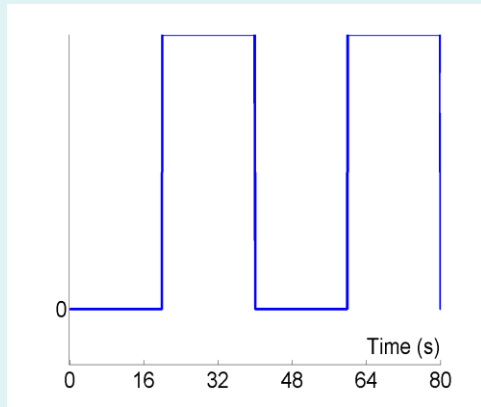
Predicted Data



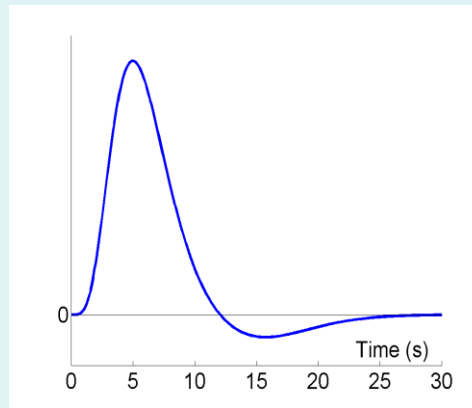
*Even more Efficient...*

# Blocked, epoch = 20s

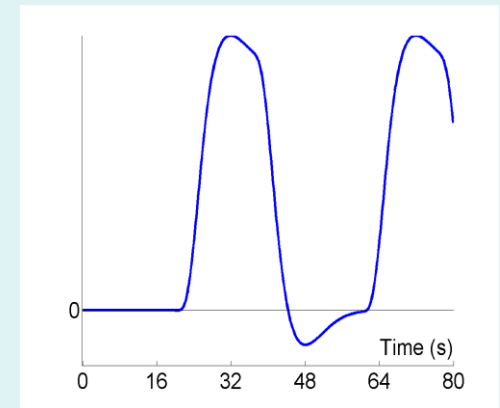
Stimulus (“Neural”)



HRF

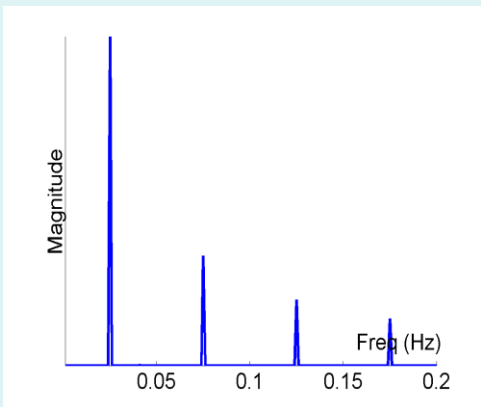


Predicted Data



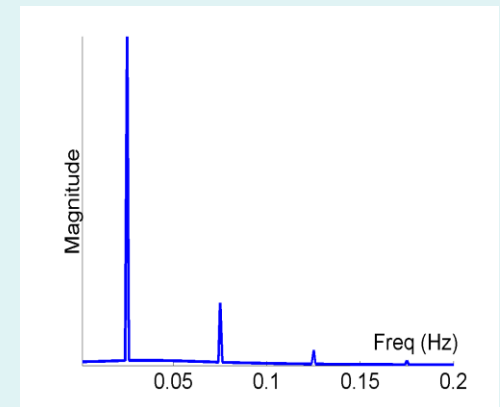
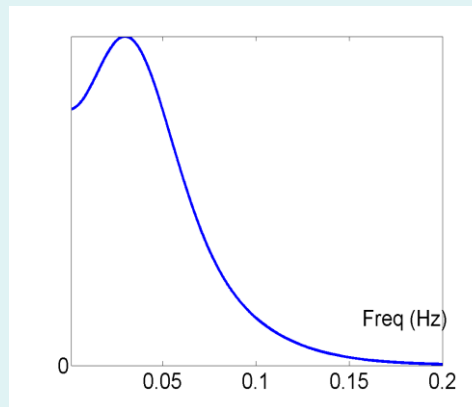
$\otimes$

$=$



$\times$

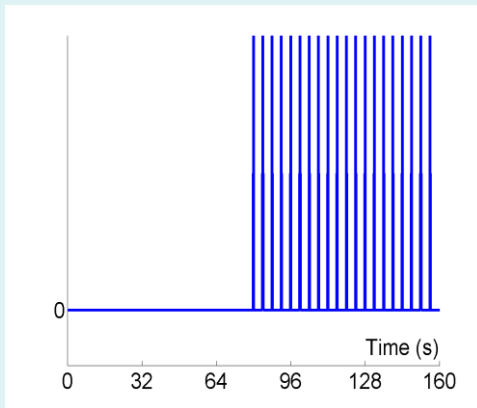
$=$



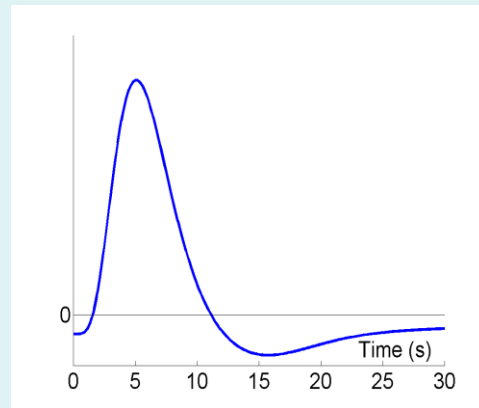
*Blocked-epoch (with small SOA) and Time-Freq equivalences*

# Blocked (80s), $SOA_{\min}=4s$ , highpass filter = $1/120s$

Stimulus (“Neural”)



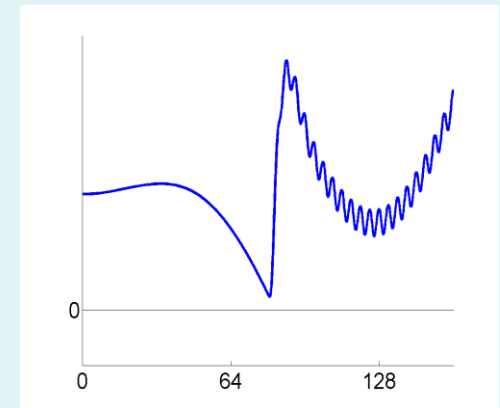
HRF



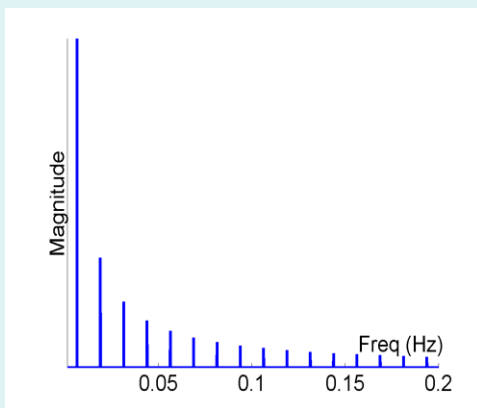
$\otimes$

=

Predicted Data

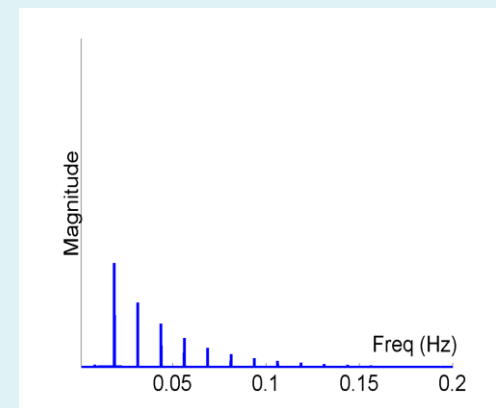
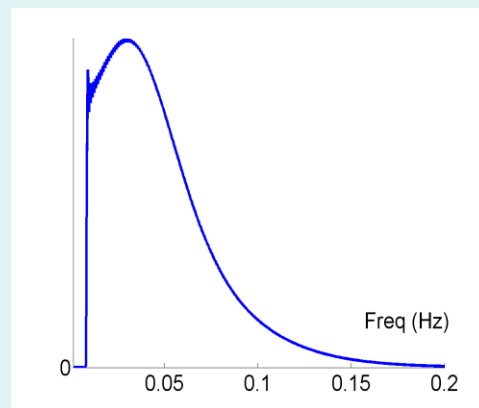


“Effective HRF” (after highpass filtering)  
(Josephs & Henson, 1999)



$\times$

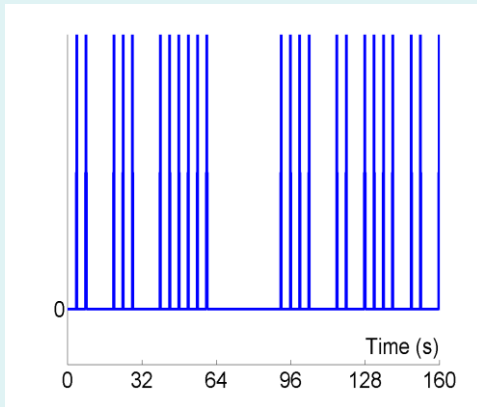
=



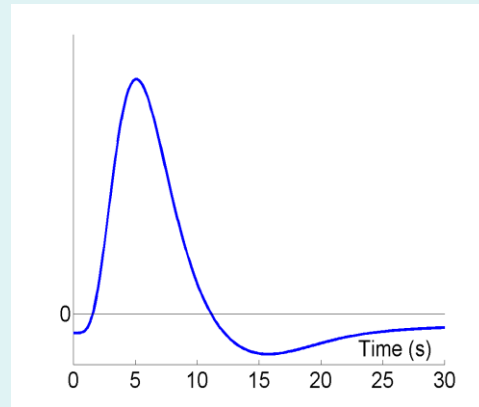
*Don't have long (>60s) blocks!*

# Randomised, $SOA_{\min}=4s$ , highpass filter = $1/120s$

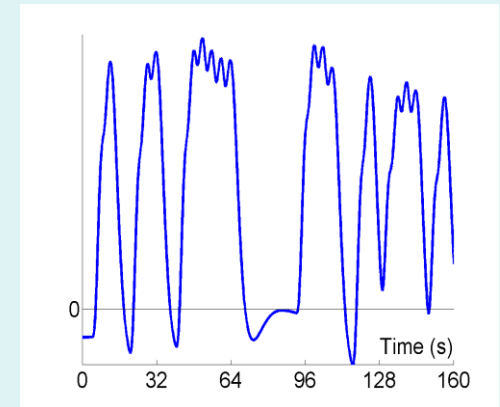
Stimulus (“Neural”)



HRF

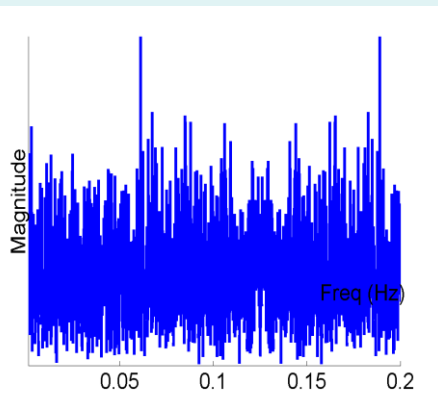


Predicted Data

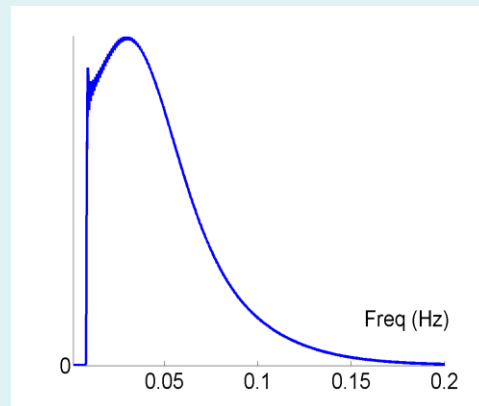


$\otimes$

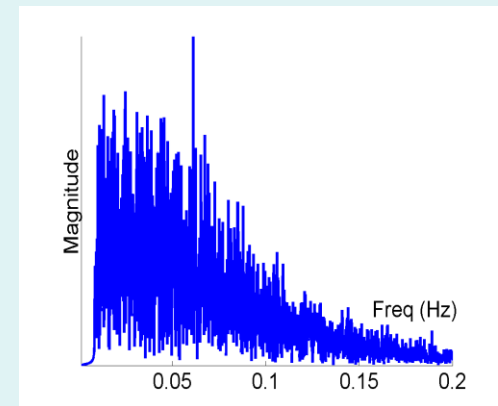
$=$



$\times$



$=$



*(Randomised design spreads power over frequencies)*

# Design Efficiency

Maximise efficiency by maximising  $t$ , by minimising the squared variance:

$$t = \frac{c^T \beta}{\sqrt{\text{var}(c^T \beta)}}$$

$X$ : design matrix  
 $c$ : contrast vector  
 $\beta$ : beta vector

Assuming that the error in our model is 'iid', each observation is drawn independently from a Gaussian distribution:

$$b \sim N(b, \underbrace{s^2 (X^T X)^{-1}}_{\text{var}(c^T b) = s^2 c^T (X^T X)^{-1} c})$$

Assuming  $\sigma$  is independent of our design, taking a fixed contrast we can only alter our design matrix to improve efficiency.

Formal definition of **design efficiency** minimises variance:

$$e \gg \frac{1}{\sqrt{c^T (X^T X)^{-1} c}}$$

Given the contrast of interest, **minimise covariance in the design matrix**

***Efficiency can be estimated before using the design***



